1. DOMÁCA ÚLOHA Z PREDMETU POČÍTAČOVÁ GRAFIKA (1)

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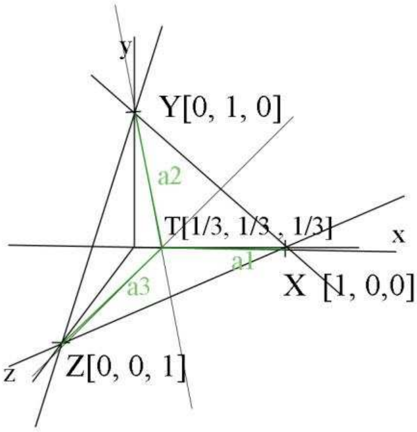
1.



$\pi : x + y + z - 1 = 0$

Dosadením do rovnice vieme určiť , že
X =[1, 0 ,0]
Y = [0, 1, 0]
Z = [0, 0, 1]

X' je stred YZ => T_x $[0, \frac{1}{2}, \frac{1}{2}]$

Y' je stred XZ => T_y $[\frac{1}{2}, 0, \frac{1}{2}]$

Z' je stred XY => T_z $[\frac{1}{2}, \frac{1}{2}, 0]$

Čiže $T = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$ $\qquad s = T - O = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$

$a_1 = T - X = (-\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$ $\qquad a_2 = T - Y = (\frac{1}{3}, -\frac{2}{3}, \frac{1}{3})$ $\qquad a_3 = T - Z = (\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$

Zoberme si bod A[x,y,z] a chceme dostat bod A' $[x_1, x_2]$

Zobrazovacie rovnice

$$x_1 = \frac{\begin{vmatrix} x - t_x & y - t_y & z - t_z \\ a_{2x} & a_{2y} & a_{2z} \\ s_x & s_y & s_z \end{vmatrix}}{\begin{vmatrix} a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \\ s_x & s_y & s_z \end{vmatrix}} = \frac{\begin{vmatrix} x - \frac{1}{3} & y - \frac{1}{3} & z - \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}}{\begin{vmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}} =$$

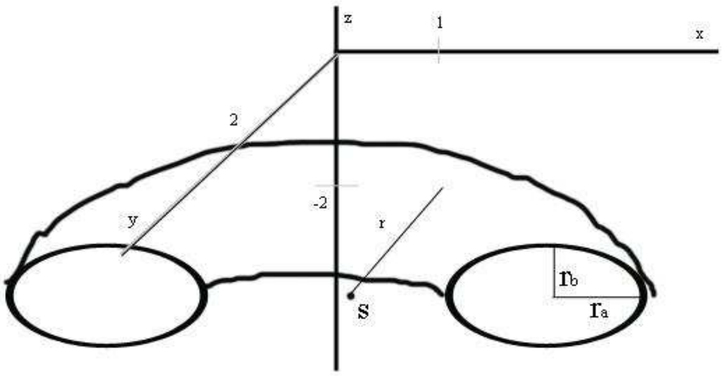
$$= \frac{-\frac{2}{9}(x-\frac{1}{3})+\frac{1}{9}(y-\frac{1}{3})+\frac{1}{9}(z-\frac{1}{3})+\frac{2}{9}(z-\frac{1}{3})-\frac{1}{9}(y-\frac{1}{3})-\frac{1}{9}(x-\frac{1}{3})}{\frac{4}{27}+\frac{1}{27}+\frac{1}{27}+\frac{2}{27}-\frac{1}{27}+\frac{2}{27}} =$$

$$= \frac{-\frac{1}{3}(x-\frac{1}{3})+\frac{1}{3}(z-\frac{1}{3})}{\frac{9}{27}} = \frac{27}{9}.\frac{1}{3}(-x+\frac{1}{3}+z-\frac{1}{3}) = z-x$$

$$x_2 = \frac{\begin{vmatrix} x-t_x & y-t_y & z-t_z \\ a_{1x} & a_{1y} & a_{1z} \\ s_x & s_y & s_z \end{vmatrix}}{-\begin{vmatrix} a_{1x} & a_{1y} & a_{1z} \\ a_{2x} & a_{2y} & a_{2z} \\ s_x & s_y & s_z \end{vmatrix}} = \frac{\begin{vmatrix} x-\frac{1}{3} & y-\frac{1}{3} & z-\frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}}{\begin{vmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix}} =$$

$$= \frac{\frac{1}{9}(x-\frac{1}{3})+\frac{1}{9}(y-\frac{1}{3})-\frac{2}{9}(z-\frac{1}{3})-\frac{1}{9}(z-\frac{1}{3})+\frac{2}{9}(y-\frac{1}{3})-\frac{1}{9}(x-\frac{1}{3})}{-\frac{9}{27}} =$$

$$= \frac{\frac{3}{9}(y-\frac{1}{3})+\frac{3}{9}(z-\frac{1}{3})}{-\frac{9}{27}} = -\frac{27}{9}.\frac{3}{9}(y-\frac{1}{3}-z+\frac{1}{3}) = z-y$$



2.

Parametrické vyjadrenie elipsy :

$x = R + r_a \cos\varphi$
$y = 0$
$z = r_b \sin\varphi \qquad \varphi \in \langle 0, \pi \rangle$

Matica otočenie okolo z :

$$R_z = \begin{pmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \psi \in \langle 0, 2\pi \rangle$$

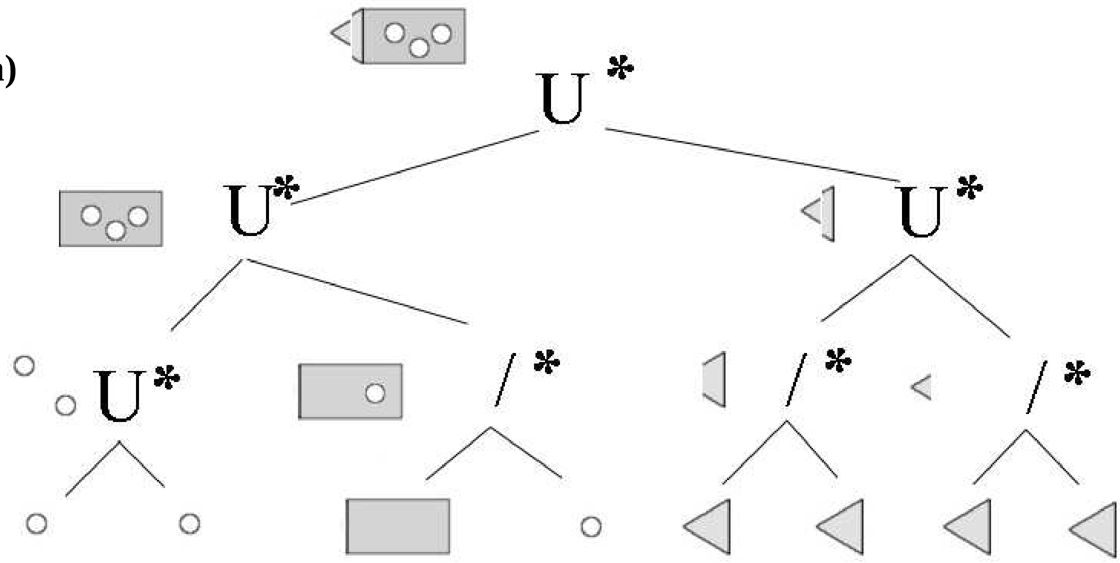
$$\begin{pmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R + r_a \cos\varphi \\ 0 \\ r_b \sin\varphi \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\psi (R + r_a \cos\varphi) \\ \sin\psi (R + r_a \cos\varphi) \\ r_b \sin\varphi \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\psi (5 + 2\cos\varphi) \\ \sin\psi (5 + 2\cos\varphi) \\ \sin\varphi \\ 1 \end{pmatrix}$$

a ešte treba posunúť o S

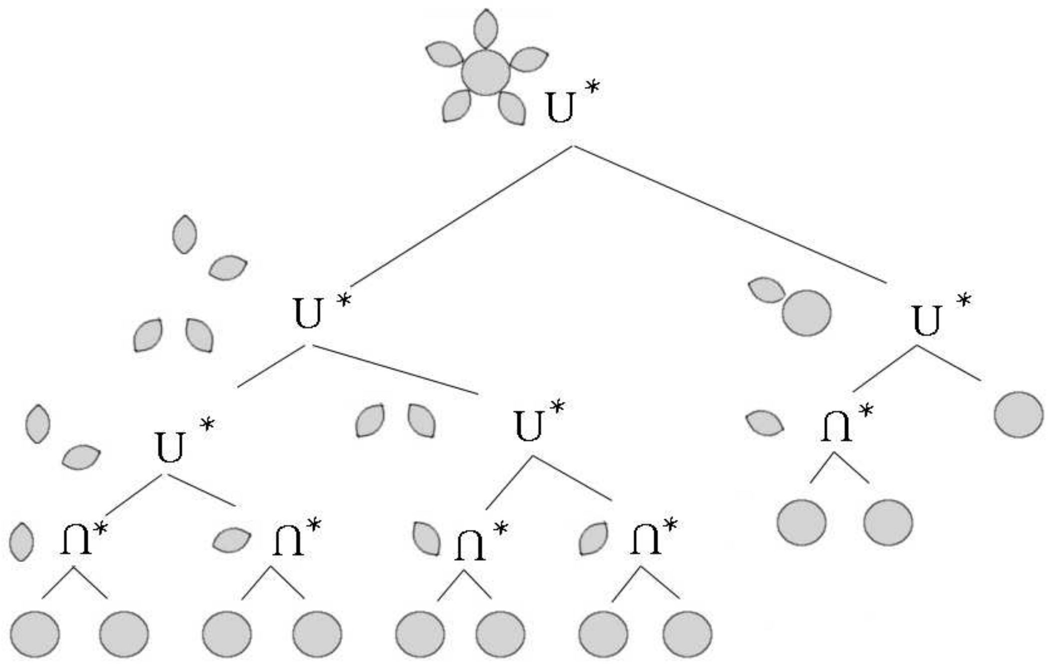
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\psi (5 + 2\cos\varphi) \\ \sin\psi (5 + 2\cos\varphi) \\ \sin\varphi \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\psi (5 + 2\cos\varphi) + 1 \\ \sin\psi (5 + 2\cos\varphi) + 2 \\ \sin\varphi - 2 \\ 1 \end{pmatrix}$$

3.

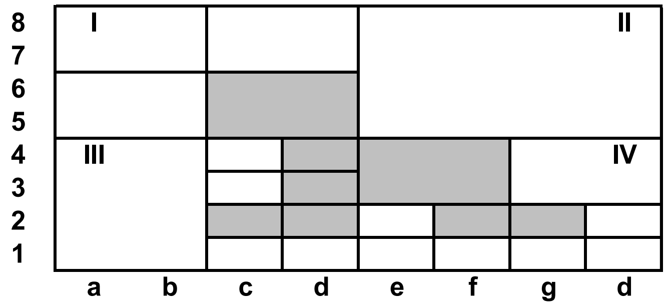
a)



b)



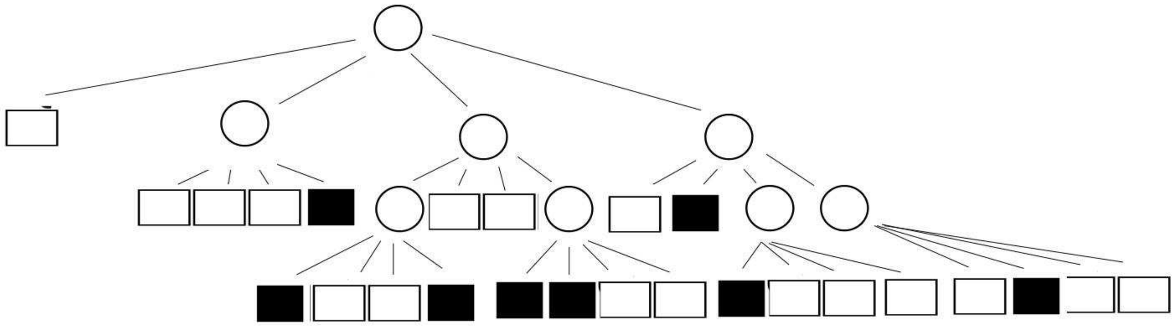
4.



Vrchol sa delí

prázdny block

Plný blok Poradie II , I, III, IV



Lineárny záznam:

B – plný blok (ten čierny)
W - prázdny blok (ten bliely)

W (WWWB) **(**(BWWB)WW(BBWW)**)** **(**WB(BWWW)(WBWW)**)**

5

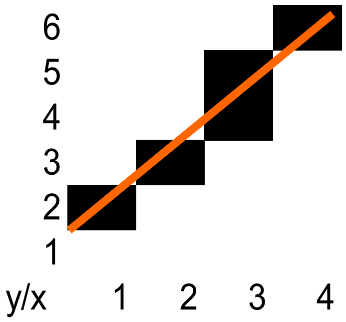
Pre body $(x_1; y_1) = (1; 2)$ a $(x_2; y_2) = (4,6)$

$\Delta x = x_2 - x_1 = 4 – 1 = 3$
$\Delta y = y_2 - y_1 = 6 – 2 = 4$
$a = 4/3$ ... určila som smernicu => $\Delta y > \Delta x$ II. oktant

$3<4 = \Delta x < \Delta y$ = > Transformácia na I. oktant : $G = 1/a = 3/4 = 0,75$
$H = 1$

$\max(\Delta x, \Delta y) = \max(3,4) = 4;$

L	*point X*	*point Y*	*X*	*Y*
1	1	2	1,75	3
2	2	3	2,5	4
3	3	4	3,25	5
4	3	5	4	6
5	4	6	4,75	7



Ide o DDA algoritmus (pre II. oktant).

DDA algoritmus pretože pozdĺž riadiacej osi sa pripočítava 1 v každom kroku pixel a na druhej osi sa pridáva vypočítaný prírastok (smernica) . Pixel sa vykresľuje sa podľa zaokrúhlených hodnôt.

6.

ROZHODOVANIE

$y_{i+1} = y_i + 1$ Riadiaca os vždy bude väčšia o 1

$$x_{i+1} \begin{cases} x_i & d1 > d2 \Rightarrow, d1 - d2 \geq 0 \\ x_i - 1 & d1 < d2 \Rightarrow d1 - d2 < 0 \end{cases}$$

Vypočítame si x-ovú súradnicu prislúchajúcu súradnici $y_{i+1} = y_i + 1$ ležiacej na úsečke AB

$$y = \frac{\Delta y}{\Delta x} x + q \qquad y_{i+1} = \frac{\Delta y}{\Delta x} x + q \Rightarrow \frac{\Delta y}{\Delta x} x = (y_i + 1) - q \Rightarrow x = [\ (y_i + 1) - q\ ] \frac{\Delta x}{\Delta y}$$

Pre d_1 a d_2 teda platí že $x_{i+1} < x < x_i$ a teda

$$x_i - 1 + d_1 = x \Rightarrow d_1 = x - (x_i - 1)$$
$$x + d_2 = x_i \Rightarrow d_2 = x_i - x$$

ODVODENIE ALGORITMU

Dosadením x do vzťahu pre výpočet d1 a d2 dostávame

$$d_1 = [(y_i + 1) - q] \frac{\Delta x}{\Delta y} - (x_i - 1)$$

$$d_2 = x_i - [(y_i + 1) - q] \frac{\Delta x}{\Delta y}$$

$$\Delta d = d_1 - d_2 = \left\{\left[(y_i + 1) - q\right]\frac{\Delta x}{\Delta y} - (x_i - 1)\right\} - \left\{x_i - \left[(y_i + 1) - q\right]\frac{\Delta x}{\Delta y}\right\} =$$

$$\left[(y_i + 1) - q\right]\frac{\Delta x}{\Delta y} - (x_i - 1) - x_i + \left[(y_i + 1) - q\right]\frac{\Delta x}{\Delta y} = -x_i + 1 - x_i + 2\frac{\Delta x}{\Delta y}\left[(y_i + 1) - q\right] =$$

$$1 - 2x_i + 2\frac{\Delta x}{\Delta y}\left[(y_i + 1) - q\right]$$

Prenásobím Δy v tomto prípade ma Δy kladne znamienko => znamienká nebudú meniť.

$$\Delta d \Delta y = \Delta y - 2\Delta y x_i + 2\Delta x\left[(y_i + 1) - q\right] = \Delta y - 2\Delta y x_i + 2\Delta x(y_i + 1) - 2\Delta x q = -2\Delta y x_i + 2\Delta x(y_i + 1) + C$$
$$C = \Delta y - 2\Delta x q$$

$\Delta d \Delta y$ budem označovať ako p_i

$$p_i = -2\Delta y x_i + 2\Delta x(y_i + 1) + C$$

Teraz pomocou p_i vyjadrím p_{i+1}

$$p_{i+1} = -2\Delta y x_{i+1} + 2\Delta x(y_{i+1} + 1) + C$$

Vyjadrím si p

$$p = p_{i+1} - p_i = \left[-2\Delta y x_{i+1} + 2\Delta x(y_{i+1} + 1) + C\right] - \left[-2\Delta y x_i + 2\Delta x(y_i + 1) + C\right]$$
$$p = -2\Delta y x_{i+1} + 2\Delta x(y_{i+1} + 1) + C + 2\Delta y x_i - 2\Delta x(y_i + 1) - C$$
$$p = -2\Delta y x_{i+1} + 2\Delta x(y_{i+1} + 1) + 2\Delta y x_i - 2\Delta x(y_i + 1)$$

Využijem fakt že $y_{i+1} = y_i + 1$

$$p = p_{i+1} - p_i = -2\Delta y x_{i+1} + 2\Delta x(y_i + 1 + 1) + 2\Delta y x_i - 2\Delta x(y_i + 1) =$$
$$-2\Delta y x_{i+1} + 2\Delta x(y_i + 2) + 2\Delta y x_i - 2\Delta x(y_i + 1) = 2\Delta x(y_i + 2 - y_i - 1) - 2\Delta y x_{i+1} + 2\Delta y x_i =$$
$$2\Delta x - 2\Delta y x_{i+1} + 2\Delta y x_i$$

Dostávam teda že

Ak $x_{i+1} = x_i \Rightarrow 2\Delta x - 2\Delta y x_{i+1} + 2\Delta y x_i = 2\Delta x - 2\Delta y x_i + 2\Delta y x_i = 2\Delta x = c_1$

Ak $x_{i+1} = x_i - 1 \Rightarrow 2\Delta x - 2\Delta y(x_i - 1) + 2\Delta y x_i = 2\Delta x - 2\Delta y(x_i - 1 - x_i) = 2\Delta x + 2\Delta y = c_2$

Inicializácia p do rovnice úsečky dosadím začiatočný bod A so súradnicami $[x_0; y_0]$

$$p_0 = -2\Delta y x_0 + 2\Delta x(y_0 + 1) + \Delta y - 2\Delta x q$$
$$p_0 = -2\Delta y x_0 + 2\Delta x(\frac{\Delta y}{\Delta x} x_0 + q + 1) + \Delta y - 2\Delta x q$$
$$p_0 = -2\Delta y x_0 + 2\Delta y x_0 + 2\Delta x q - 2\Delta x q + 2\Delta x + \Delta y$$
$$p_0 = 2\Delta x + \Delta y$$

Čiže rozhodovacie pravidlo je

Ak $p_i > 0$ => Kreslím bod $[x_i, y_i+1]$ a $p_{i+1} = p_i + c_1 = p_i + 2\Delta x$

Ak $p_i < 0$ => Kreslím bod $[x_i - 1, y_i+1]$ a $p_{i+1} = p_i + c_2 = p_i + 2\Delta x + 2\Delta y$;

A [1,1] B[7,5]

$\Delta x = x_2 - x_1 = 7\text{-}1 = 6$

$\Delta y = y_2 - y_1 = 5\text{-}1 = 4$

$m = \Delta y/\Delta x = 4/6 \quad \Delta y < \Delta x \Rightarrow$ I. oktant

Tranformuje body z I oktantu do III. oktantu

[x,y] - > [-y,x]

A' = [-1,1] $\Delta x = -5+1 = -4$

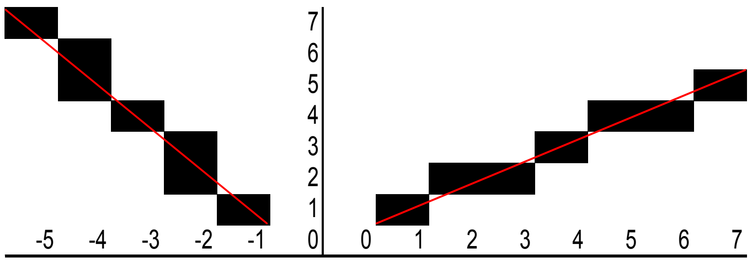
B' = [-5,7] $\Delta y = 7 - 1 = 6$

$c_1 = 2\Delta x = -8$

$c_2 = 2\Delta x + 2\Delta y = -8+12 = 4$

$p = 2\Delta x + \Delta y = -8 + 6 = -2$

Point v III	p	Point v I
-1,1	-2	1,1
-2,2	2	2,2
-2,3	-6	3,2
-3,4	-2	4,3
-4,5	2	5,4
-4,6	-6	6,4
-5,7		7,5



III oktant I. oktant