Kalman Filter

- **Published In 1960 by R.E. Kalman**
- **The Kalman filter is an efficient recursive filter that estimates the state of a dynamic system from a series of incomplete and noisy measurements.**
- **→ Kalman filter is an optimal estimator**
    - Used to estimate **system states** that can only be observed **indirectly or inaccurately** by the system itself
- **→ Kalman filter - recursive calculation!**

Dynamic linear system

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- **Model** of a dynamic linear system - used in Kalman filter
- Dynamic linear system is desribed by **State equation (1) and Output equation (2).**

X_t - State in the time t:

- (1) $X_t = A * x_{t-1} + B * u_t + w_t$

Dynamic term

A - state transition matrix

x_{t-1} - state in the time t-1

Control term

B - matrix

u_t - control signal

Noise term

W_t - process noise

Z_t - measured output:

(2) $Z_t = H * x_t + v_t$

H - measurement matrix

x_t - state in the time t

Noise term

v_t - measurement noise

- **(1)** $X_t = A * x_{t-1} + B * u_t + w_t$

  The first equation - **State equation** shows that the system state variable is dependent on the previous system state, the system control inputs and the process noise (uncertainty of the model).

- **(2)** $Z_t = H * x_t + v_t$

  The second equation **- Output equation** shows that the measured system output is dependent on the current system state and the measurement noise.

  - In special case, the measured system output could be equal to the state variable.

- Kalman filter is a set of mathematical equations
    - that provides an efficientcomputational (recursive) means to **estimate the state of a process**,

      in a way that **minimizes the mean of the squared error**.

- The random variables w_k and v_k represent the process and measurement noise (respectively).

$$p(w) \sim N(0, Q),$$

$$p(v) \sim N(0, R).$$

- They are assumed to be
  - independent (of each other),
  - white
  - with normal probability distributions
    - In practice, the process noise covariance Q and *measurement noise covariance R matrices* might *change* with each time step or measurement, however we assume they are constant.

- Q - *process noise covariance matrix*
- *R - measurement noise covariance matrix*

- Kalman filter is the estimator that satisfies two criteria:
  - 1) the expected value of the estimate should be equal to the expected value of the state
  - 2) we want to find the estimator with the smallest possible error variance

- Discrete Kalman filter **time update equations:**

$$x_k = A^* x_{(k-1)} + B^* u_{(k-1)}$$

Update Covariance:

$$P_k = A^* P_{(k-1)} A^T + Q$$

- *Discrete Kalman filter measurement update equations.*

Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$$

(2) Project the error covariance ahead

$$P_k^- = AP_{k-1}A^T + Q$$

Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$$

(3) Update the error covariance

$$P_k = (I - K_k H)P_k^-$$

Initial estimates for $\hat{x}_{k-1}$ and P_{k-1}

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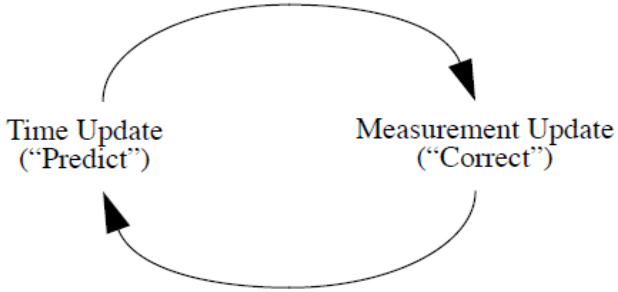
- Two groups of the equations for the Kalman filter:
  - ***time update* equations**
    - The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step.
  - and ***measurement update* equations**.
    - The measurement update equations are responsible for the feedback - for incorporating of a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.



- the time update equations -
  - ***predictor*** equations
- the measurement update equations -
  - ***corrector*** equations
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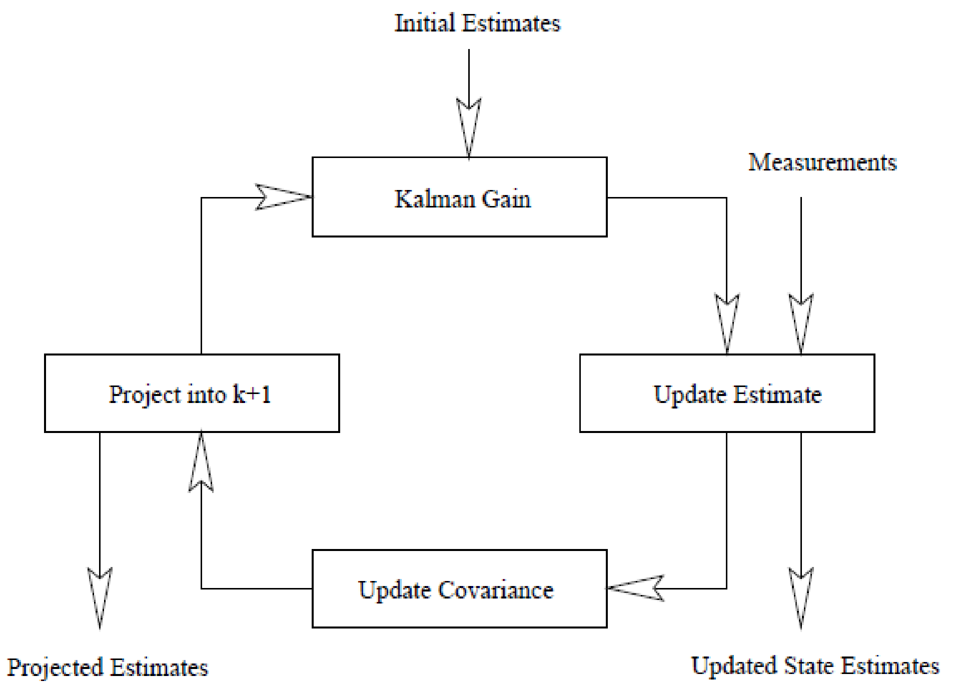
Initial Estimates
Kalman Gain
Measurements
Project into k+1
Update Estimate
Update Covariance
Projected Estimates
Updated State Estimates

- irregular noisey observations
- dynamical model of the system (matrices T, B ,H) to describe the state over the time