Matematická Morfológia

Matematická morfológia

η μορφή = forma, tvar
ο λόγος (τα λόγια) = slovo (slová)

Morfológia = štúdium formy a štruktúry (zvieratá, rastliny)

Matematická morfológia = nástroj na popis komponentov obrazu, tvaru, štruktúry
Základ – teória množín

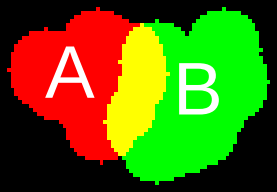
Použitie

predspracovanie
- filtrovanie šumu, zjednodušenie tvarov, ...

segmentácia
- watershed, hrany, obrys, …

popis štruktúry objektov
- kostra, konvexný obal, ...

kvantitatívny popis
- analýza tvaru (area, perimeter, ...),
- granulometria, súvislé oblasti ...

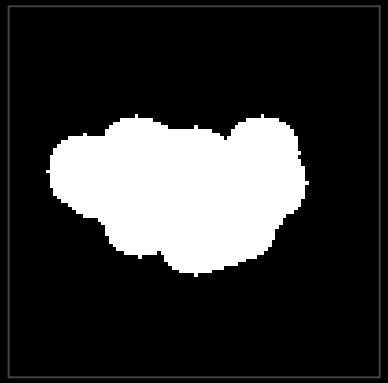


Základné definície

- Prvok množiny $x \in A$
- Prvok nepatriaci množine $x \notin A$
- Prázdna množina – neobsahuje žiadny prvok $\varnothing$
- Disjunktné množiny, ak $A \cap B = \varnothing$
- Podmnožina $A \subseteq B \Leftrightarrow (x \in A \Rightarrow x \in B)$

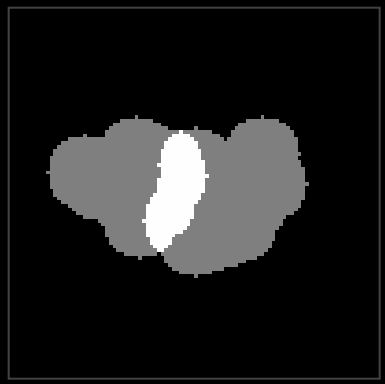
Zjednotenie

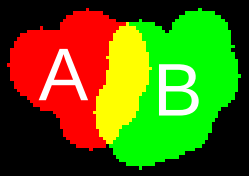
$A \cup B = \{x \mid x \in A \vee x \in B\}$



Prienik

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

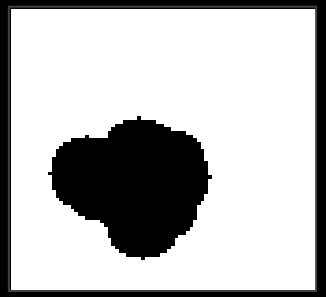




Množinové operácie

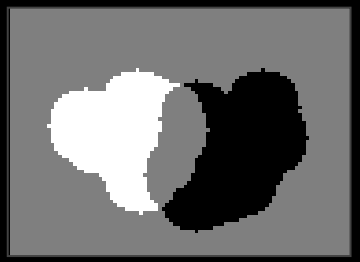
Komplement

$A^c = \{x \mid x \notin A\}$



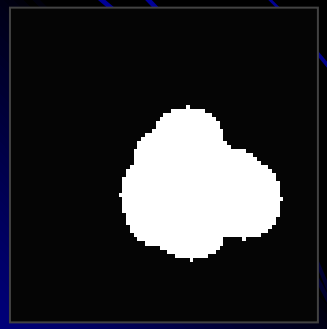
Rozdiel

$A - B = A \cap B^c = \{x \mid x \in A \wedge x \notin B\}$



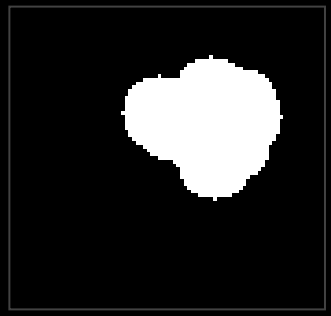
Zrkadlenie

$\hat{A} = \{x \mid x = -a, \forall a \in A\}$



Posunutie

$A_z = A + z = \{x \mid x = a + z, \forall a \in A\}$



Morfologické operácie

Vzťah:

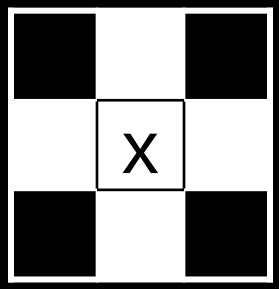
obraz (množina) – štrukturálny prvok

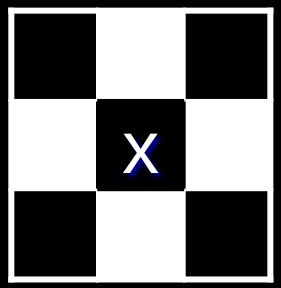
Výsledok:

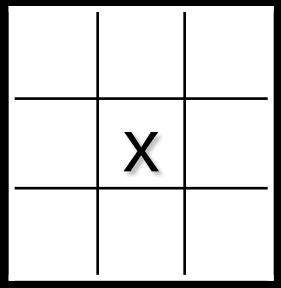
zmena, zmenšenie, zväčšenie množiny

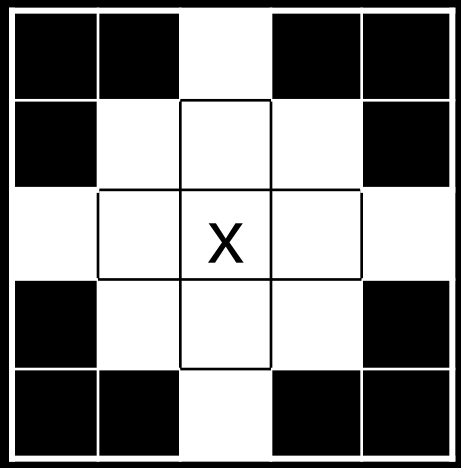
Binárna morfológia

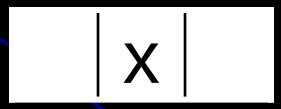
Štrukturálny prvok











x – počiatok súr. sústavy (0,0)

referenčný bod

Štrukturálny prvok

tvar
veľkosť
orientácia
pozícia vzhľadom k **x**

Závisia od aplikácie, ovplyvňujú výsledok

ŠP je (zvyčajne) oveľa menší ako obraz

Dilatácia


Minkowski

Minkowského súčet $\oplus$

$$A \oplus B = \bigcup_{b \in B} A_b$$

$$A \oplus B = \bigcup_{b \in B} \{a + b \mid a \in A\}$$

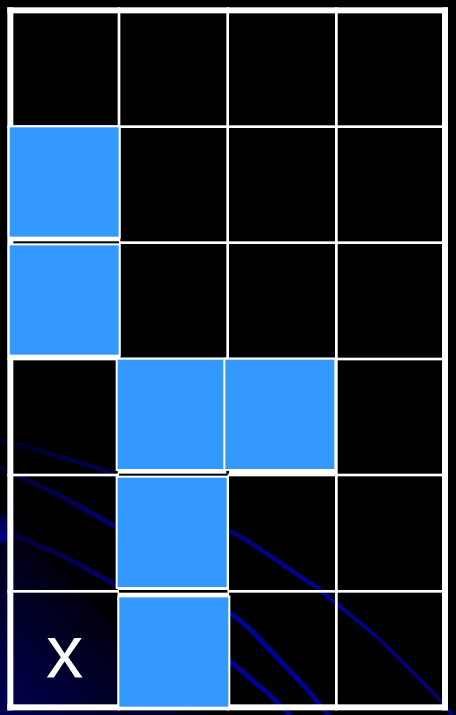
$$= \{a + b \mid a \in A,\ b \in B\}$$

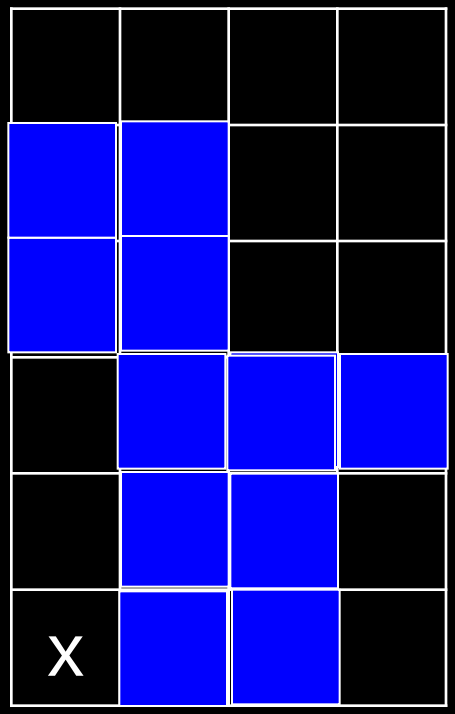
$$A \oplus B = \left\{ x \mid \hat{B}_x \cap A \neq \varnothing \right\}$$

Expanzívna operácia – zväčšuje množinu

$$A \oplus B = \bigcup_{b \in B} A_b$$

Dilatácia





B = x

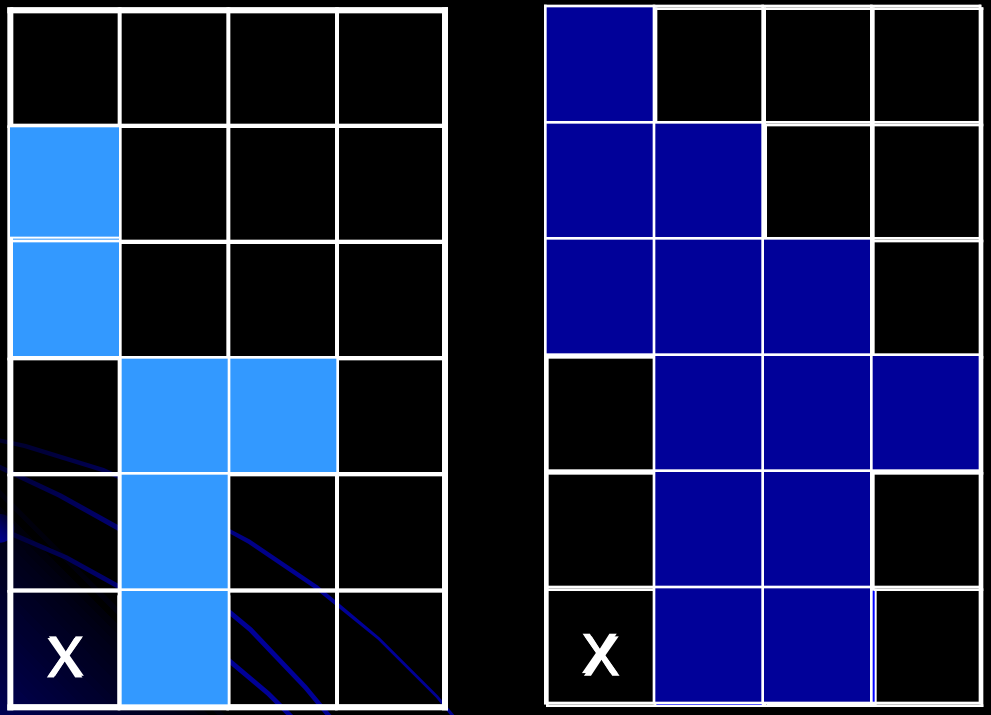
b_1 b_2

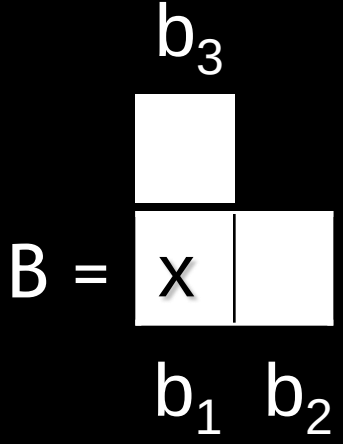
(0,0), (1,0)

(0,4), (0,3), (1,2), (2,2), (1,1), (1,0)

$$A \oplus B = \bigcup_{b \in B} A_b$$

Dilatácia



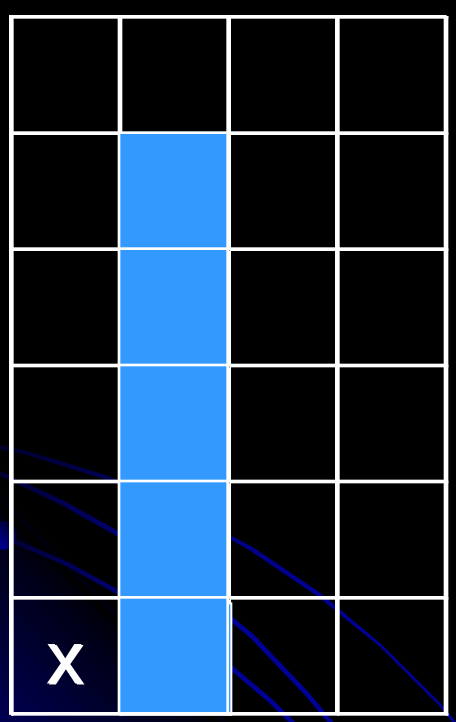


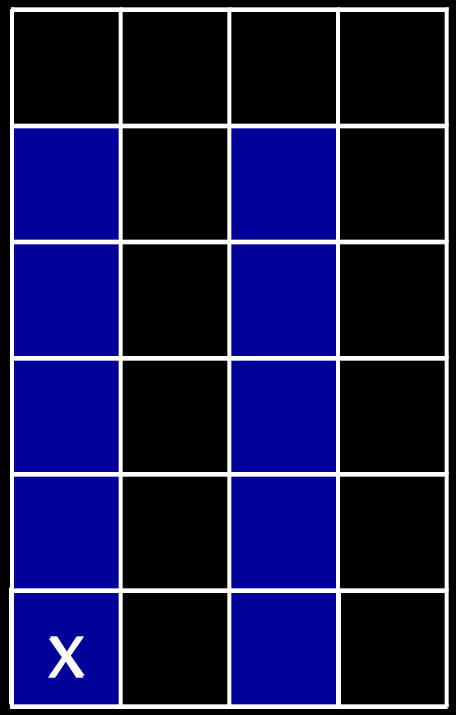
(0,0), (1,0), (0,1)

(0,4), (0,3), (1,2), (2,2), (1,1), (1,0)

$$A \oplus B = \bigcup_{b \in B} A_b$$

Dilatácia





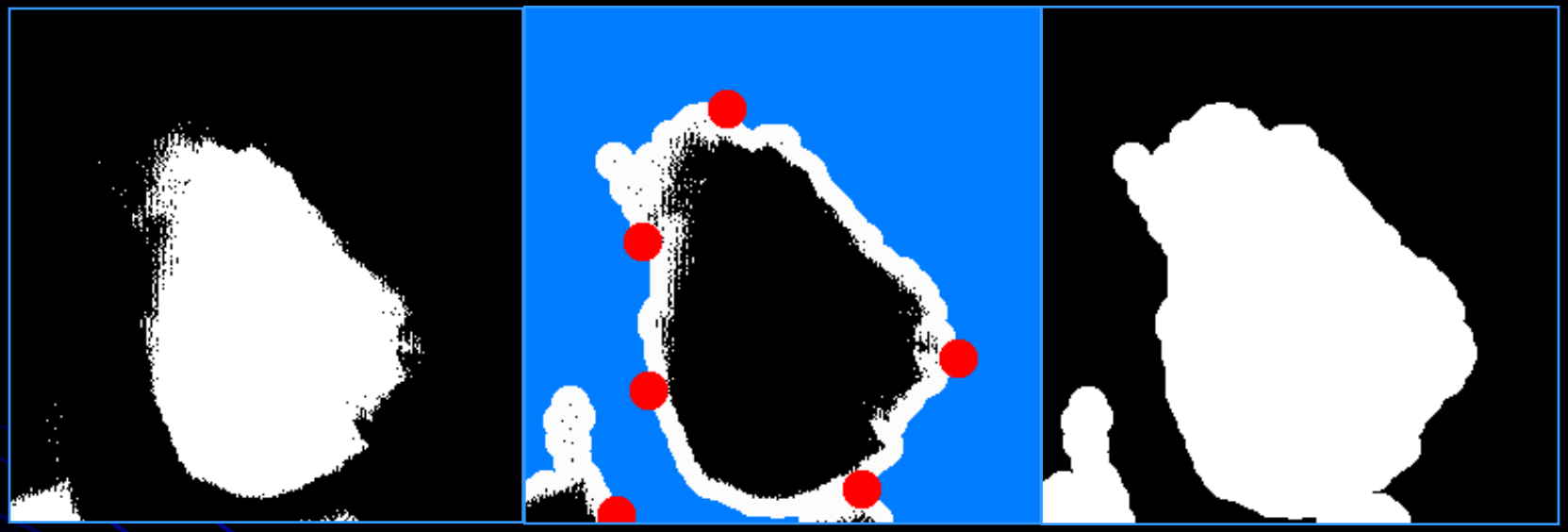
B = 

b_1 b_2

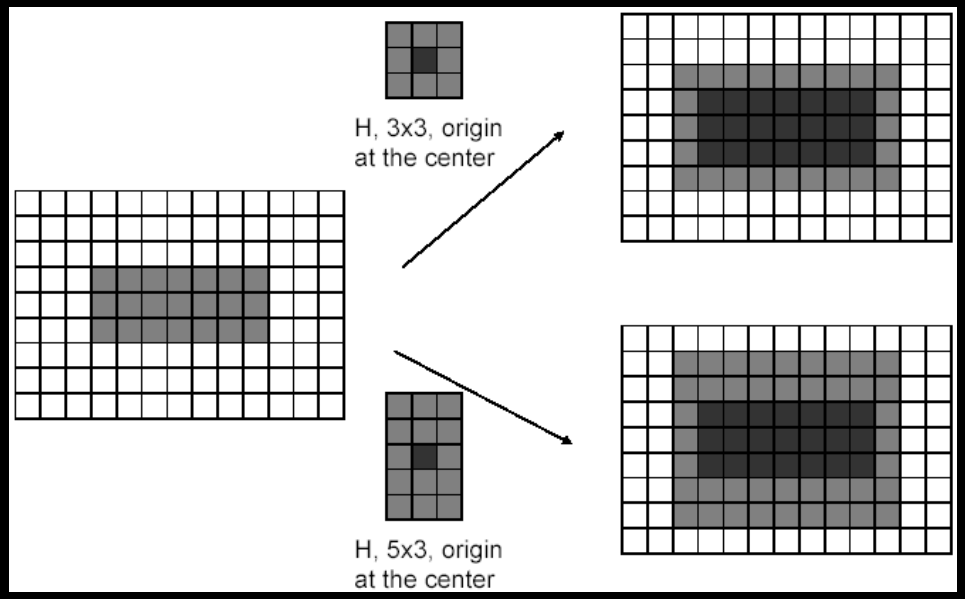
(-1,0), (1,0)

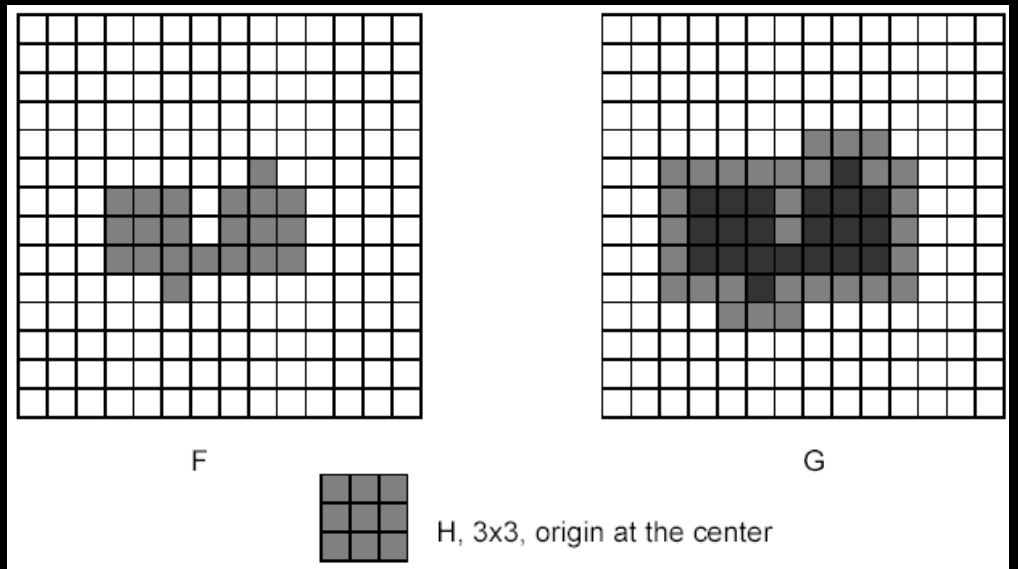
(1,0), (1,1), (1,2), (1,3), (1,4)

Dilatácia



Dilatácia


H, 3x3, origin
at the center
H, 5x3, origin
at the center


F
G
H, 3x3, origin at the center

Vlastnosti dilatácie

$$A \oplus B = B \oplus A$$

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

$$A_1 \subseteq A_2 \Rightarrow (A_1 \oplus B) \subseteq (A_2 \oplus B)$$

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$

Erózia

Minkowského rozdiel $\ominus$

$$A \ominus B = \bigcap_{b \in B} A_{-b}$$

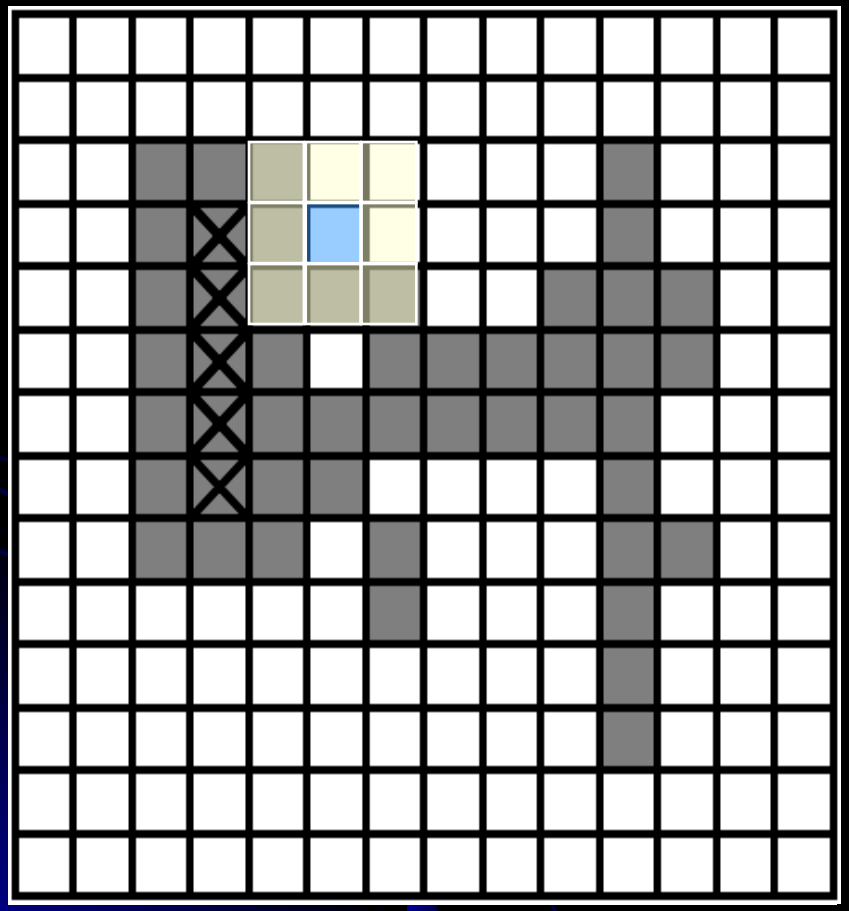
$$A \ominus B = \bigcap_{b \in B} \{a - b \mid a \in A\}$$

$$A \ominus B = \{x \mid B_x \subseteq A\}$$

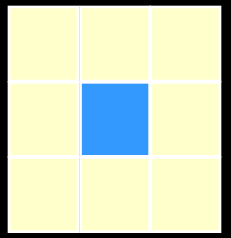
Kontraktívna operácia – zmenšuje množinu

$$A \ominus B = \{ x \mid B_x \subseteq A \}$$

Erózia

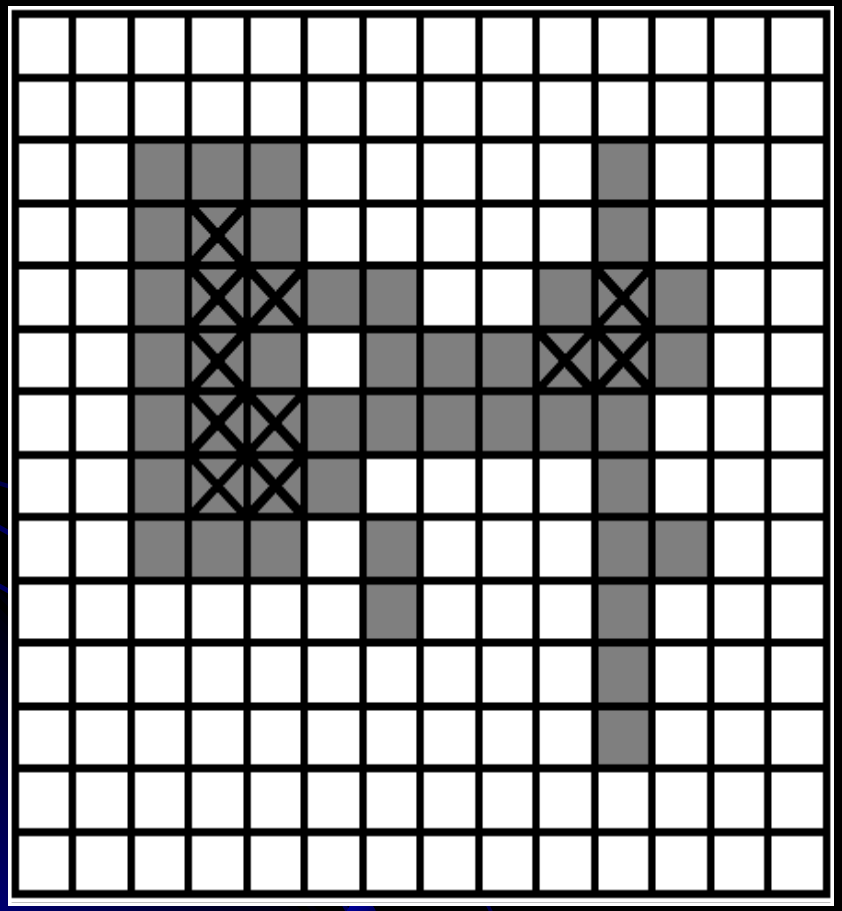


SE=

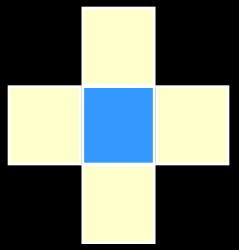


$$A \ominus B = \{ x \mid B_x \subseteq A \}$$

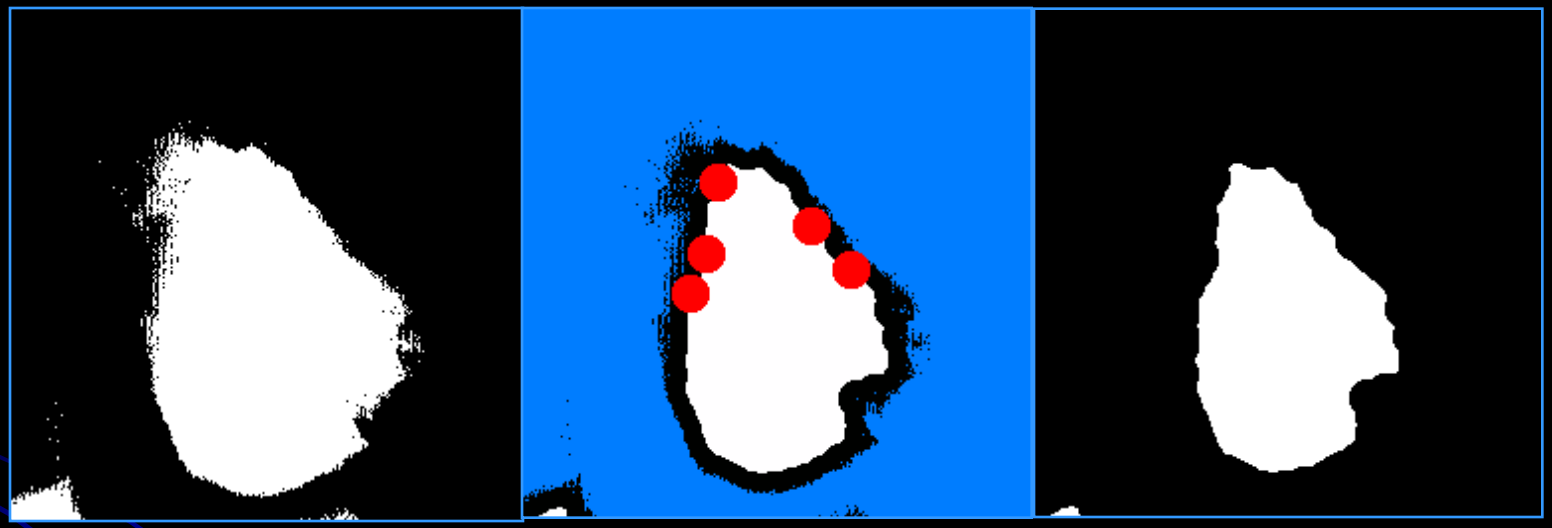
Erózia



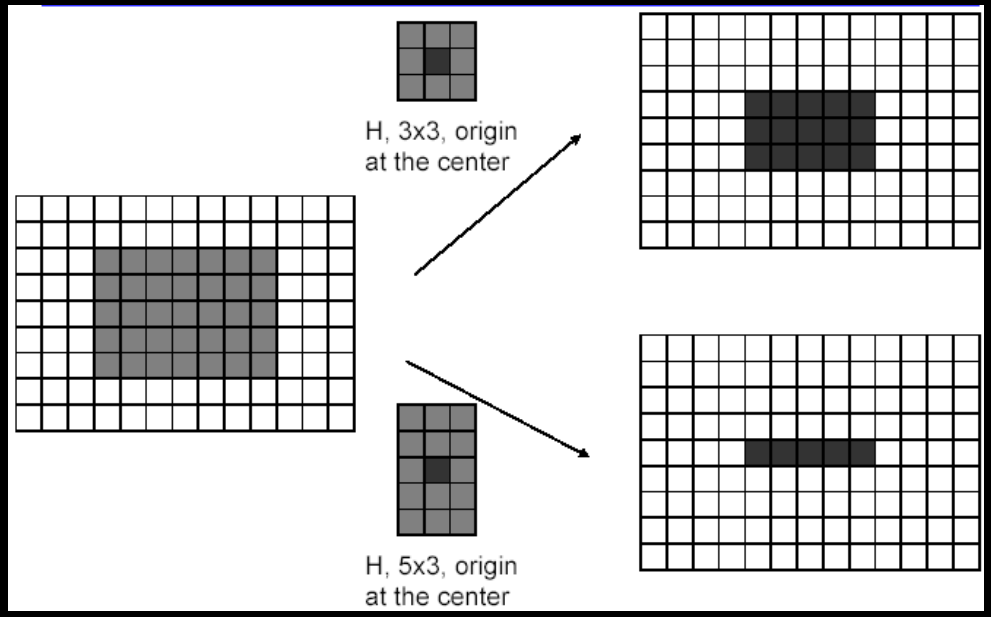
SE=

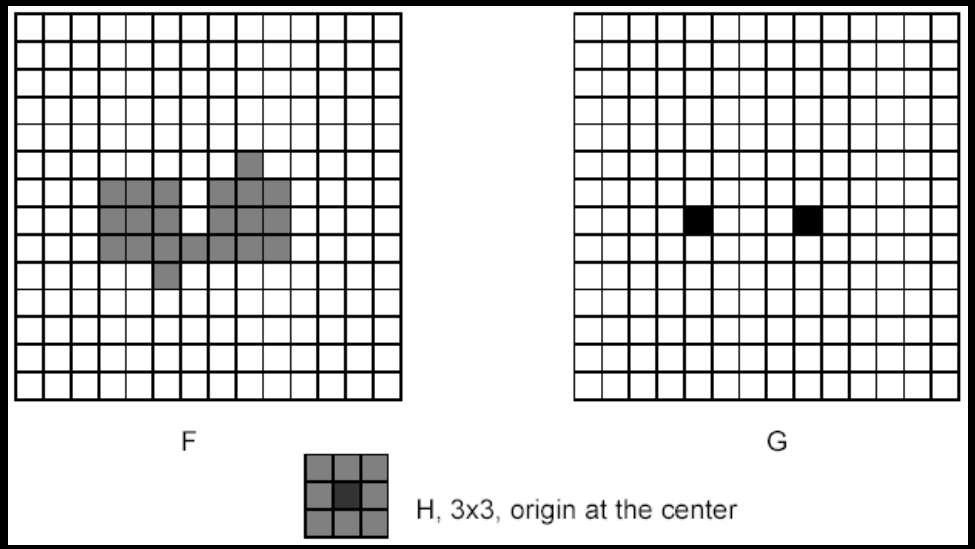


Erózia



Erózia


H, 3x3, origin
at the center
H, 5x3, origin
at the center


F
G
H, 3x3, origin at the center

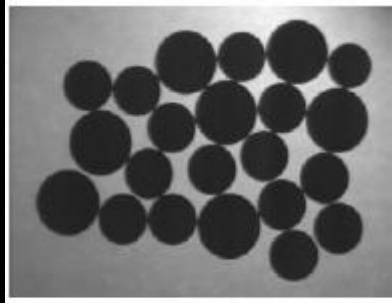
Vlastnosti erózie

$$A \ominus B \neq B \ominus A$$

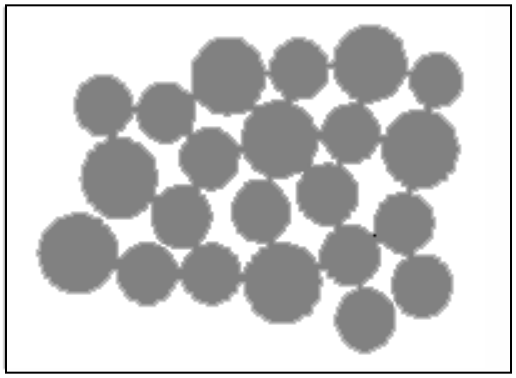
$$A_1 \subseteq A_2 \Rightarrow (A_1 \ominus B) \subseteq (A_2 \ominus B)$$
$$B_1 \subseteq B_2 \Rightarrow (A \ominus B_1) \supseteq (A \ominus B_2)$$
$$A \ominus (B \cup C) = (A \ominus B) \cap (A \ominus C)$$
$$(A \ominus B) \ominus C = A \ominus (B \oplus C)$$

$$(A \ominus B) \oplus B \subseteq A \subseteq (A \oplus B) \ominus B$$

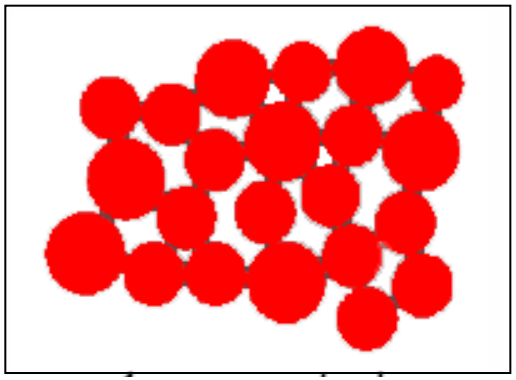
Príklad použitia



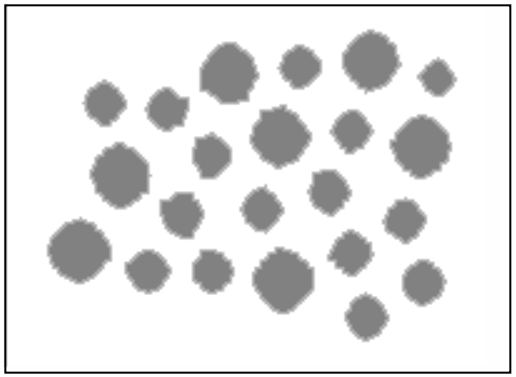
Original



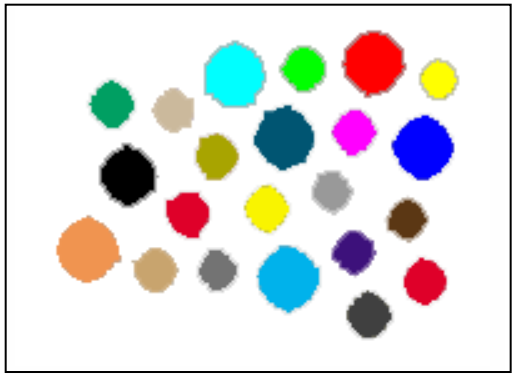
Prahovanie



1 spojitý útvar



Erózia



22 spojitých útvarov

Erózia

Originál Erodovaný 1x Erodovaný 2x

Opakovanie

dilatácia

Zväčšuje množinu

Vypĺňa diery, zálivy určitej veľkosti a tvaru

erózia

Zmenšuje množinu

Odstraňuje štruktúry určitej veľkosti a tvaru

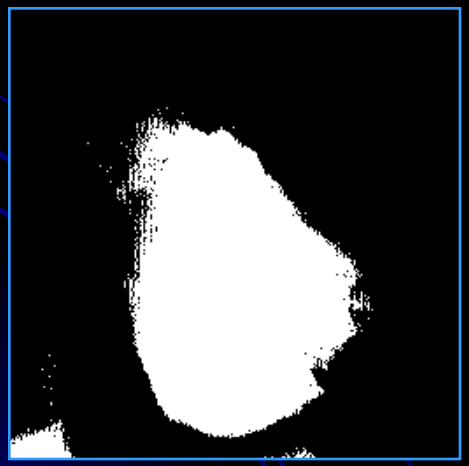
Môže rozdeliť množinu

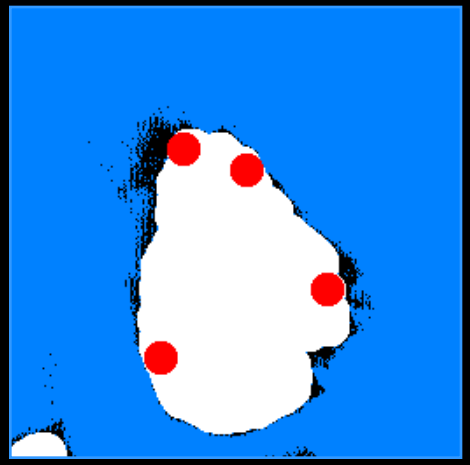
– v závislosti na štrukturálnom prvku

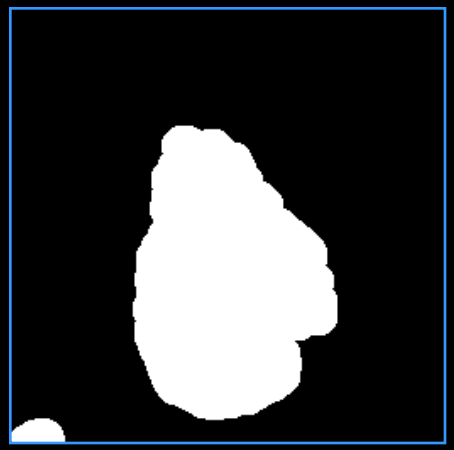
Interaktívne na http://dip.sccg.sk/

Otvorenie

$$A \circ B = (A \ominus B) \oplus B$$







Vlastnosti otvorenia

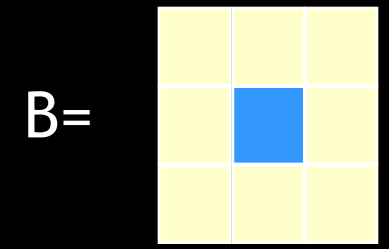
$$A \circ B \subseteq A$$

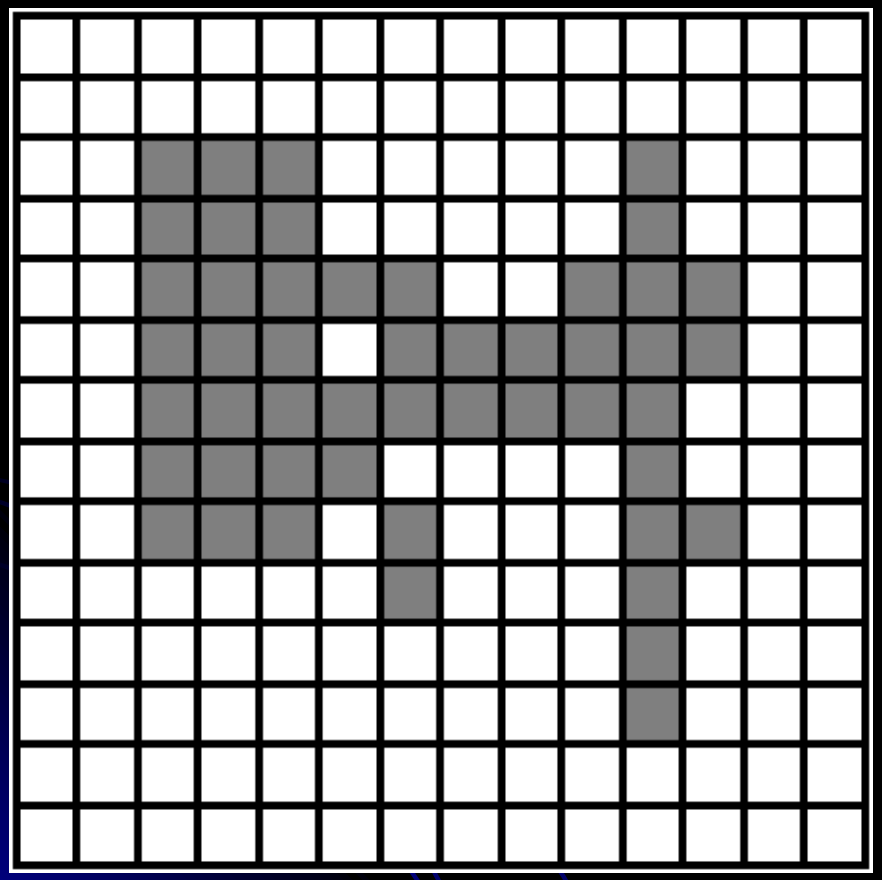
$$(A \circ B) \circ B = A \circ B$$

$$A \circ B = (A \ominus B) \oplus B$$

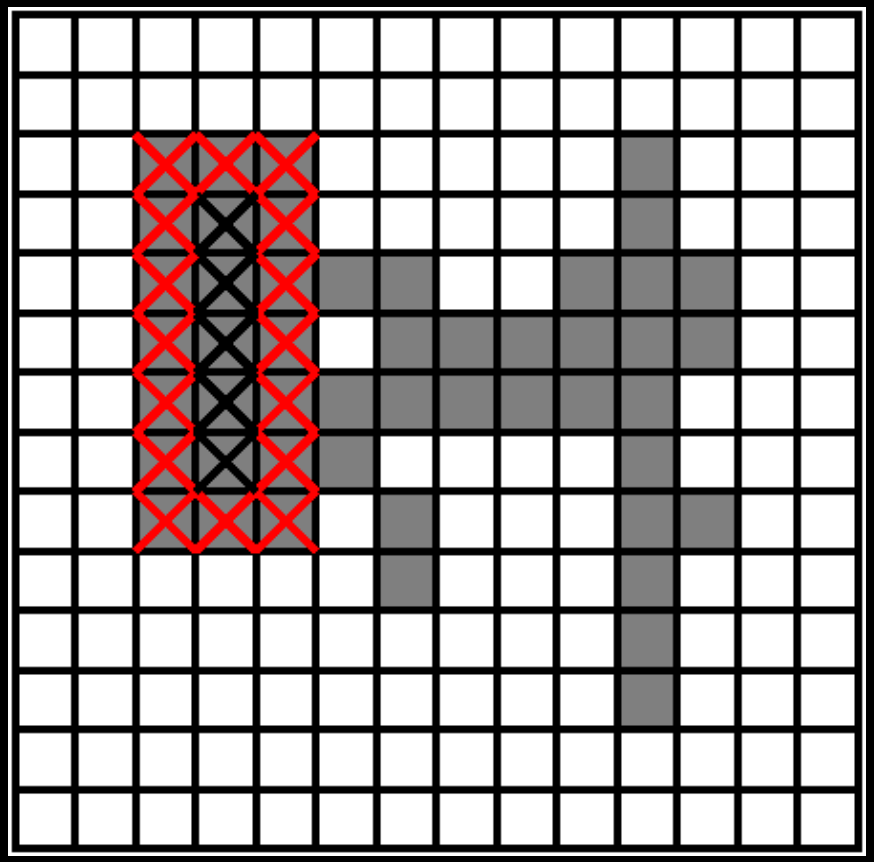
Otvorenie

B=





A

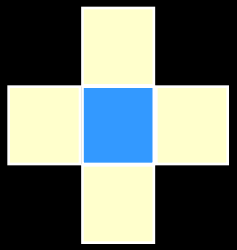


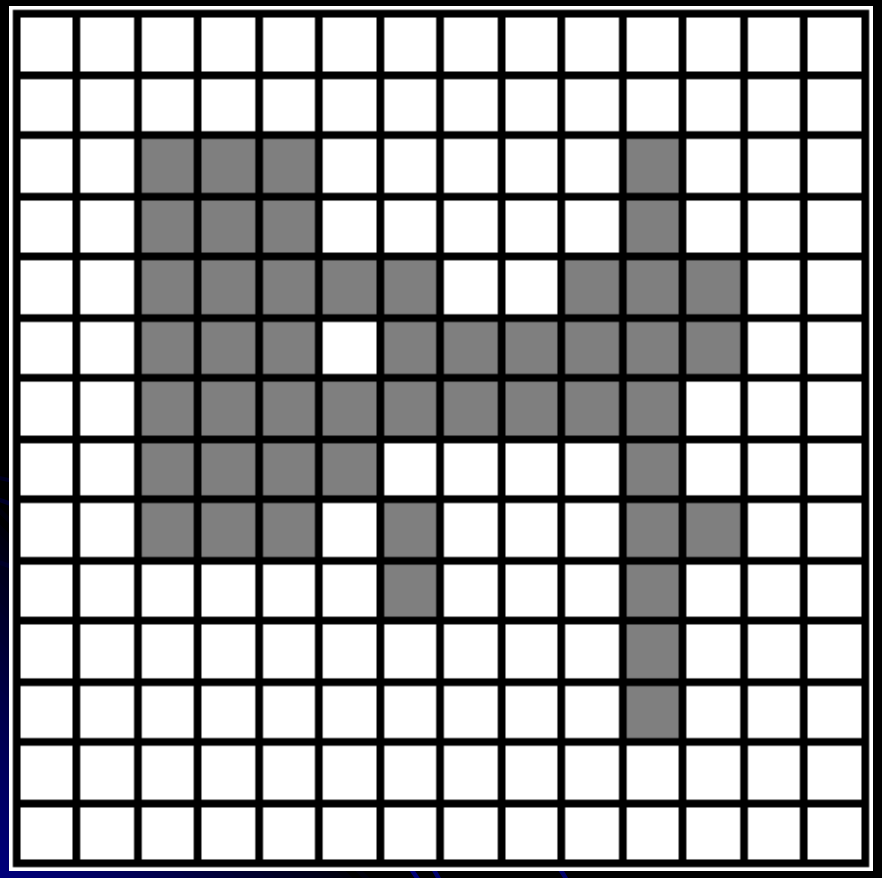
A⊖B A∘B

$$A \circ B = (A \ominus B) \oplus B$$

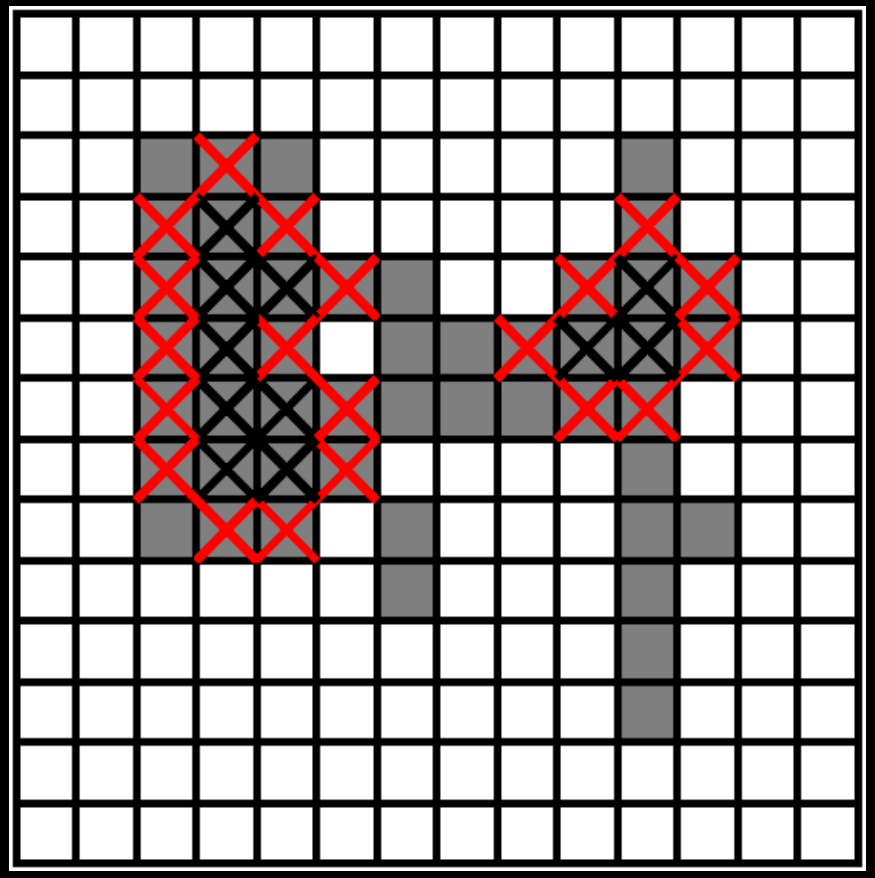
Otvorenie

B=



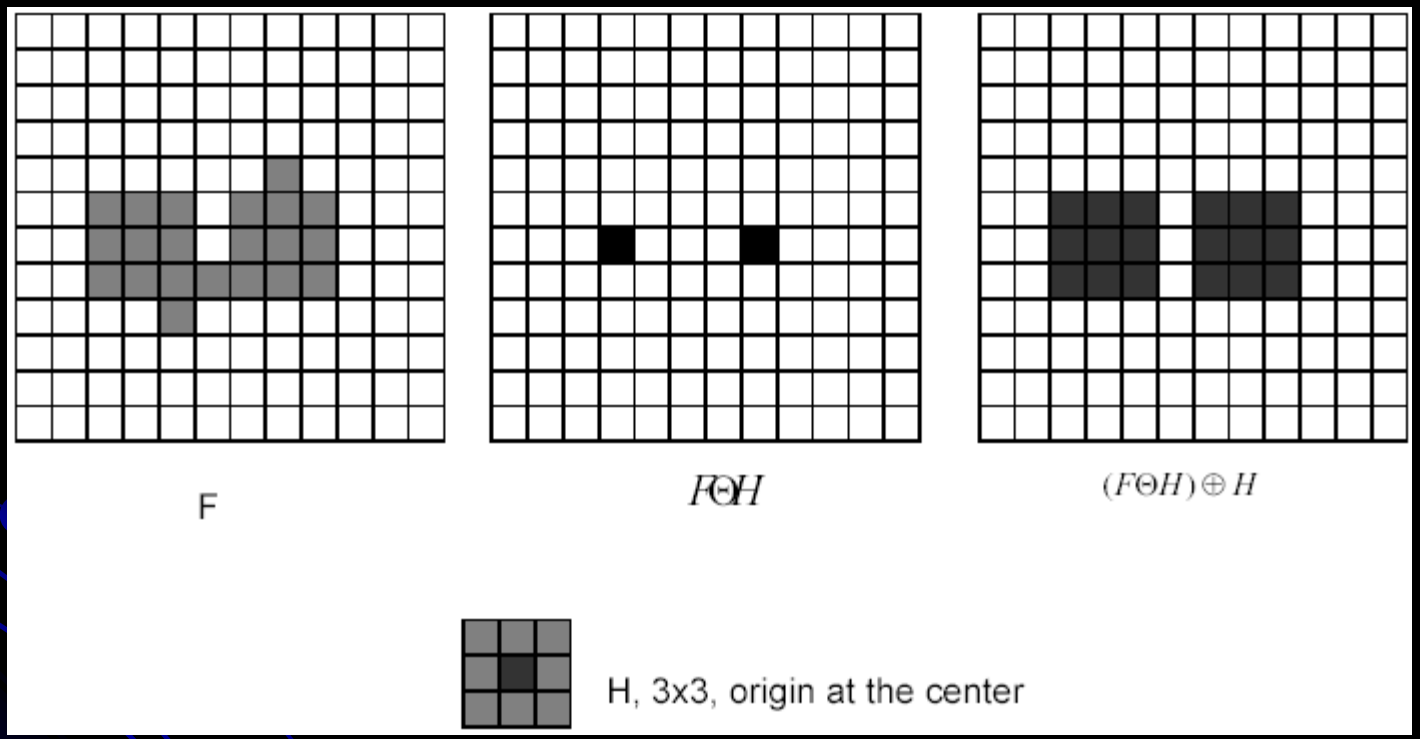


A



A⊖B A∘B

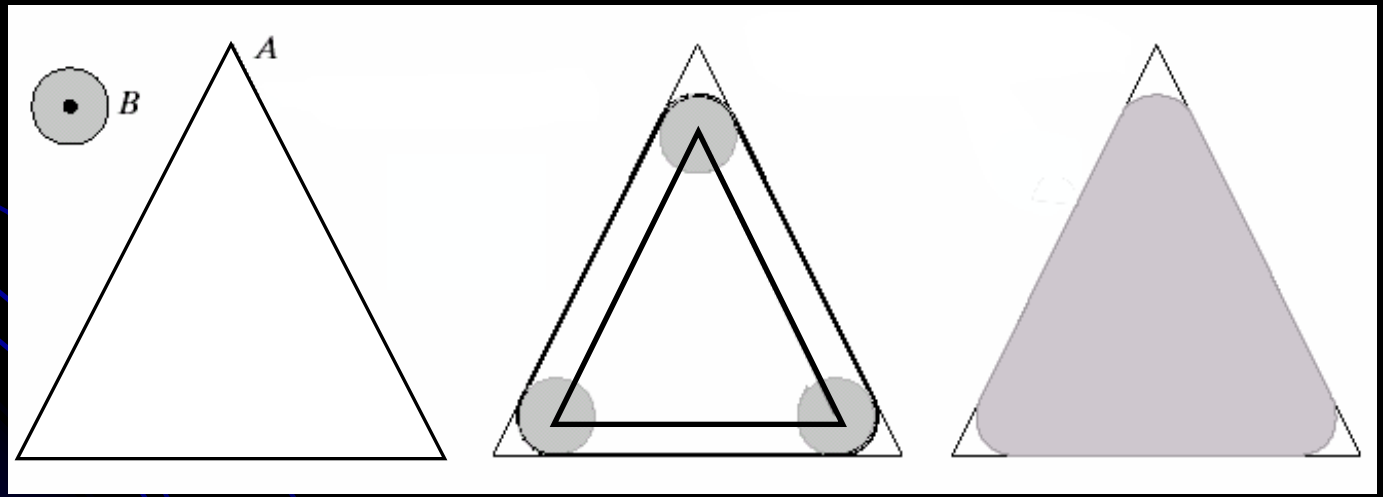
Otvorenie



Otvorenie

$$A \circ B = \bigcup \{ B_x \mid B_x \subseteq A \}$$

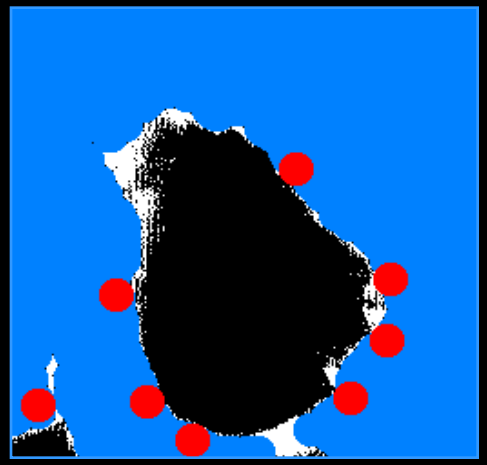
posúvame B **po vnútornej strane hranice** A



$A \ominus B$ $\oplus B$ $A \circ B$

Uzavretie

$$A \bullet B = (A \oplus B) \ominus B$$



Vlastnosti uzavretia

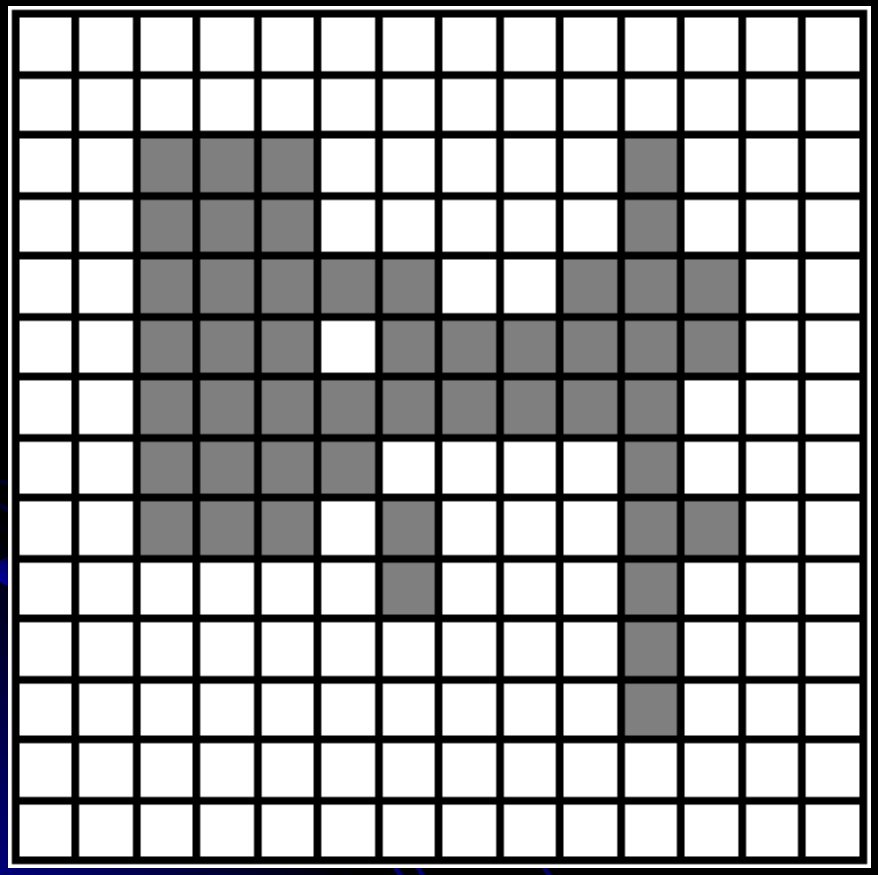
$$A \subseteq A \bullet B$$

$$(A \bullet B) \bullet B = A \bullet B$$

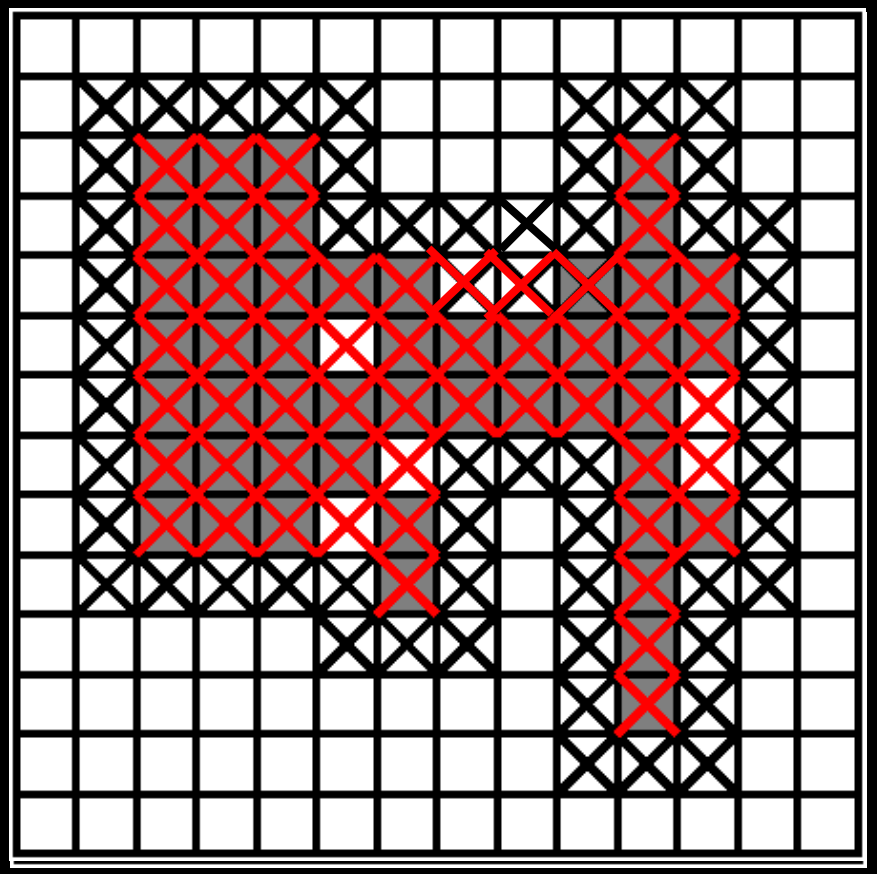
$A \bullet B = (A \oplus B) \ominus B$

Uzavretie

B=



A

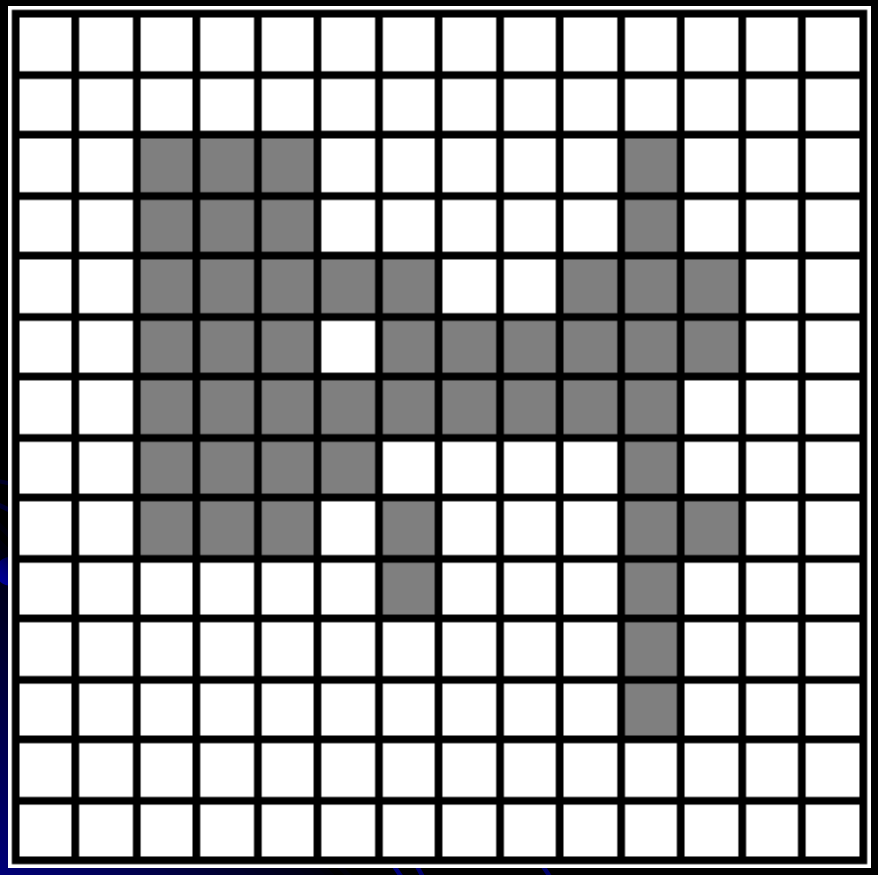


A⊕B A • B

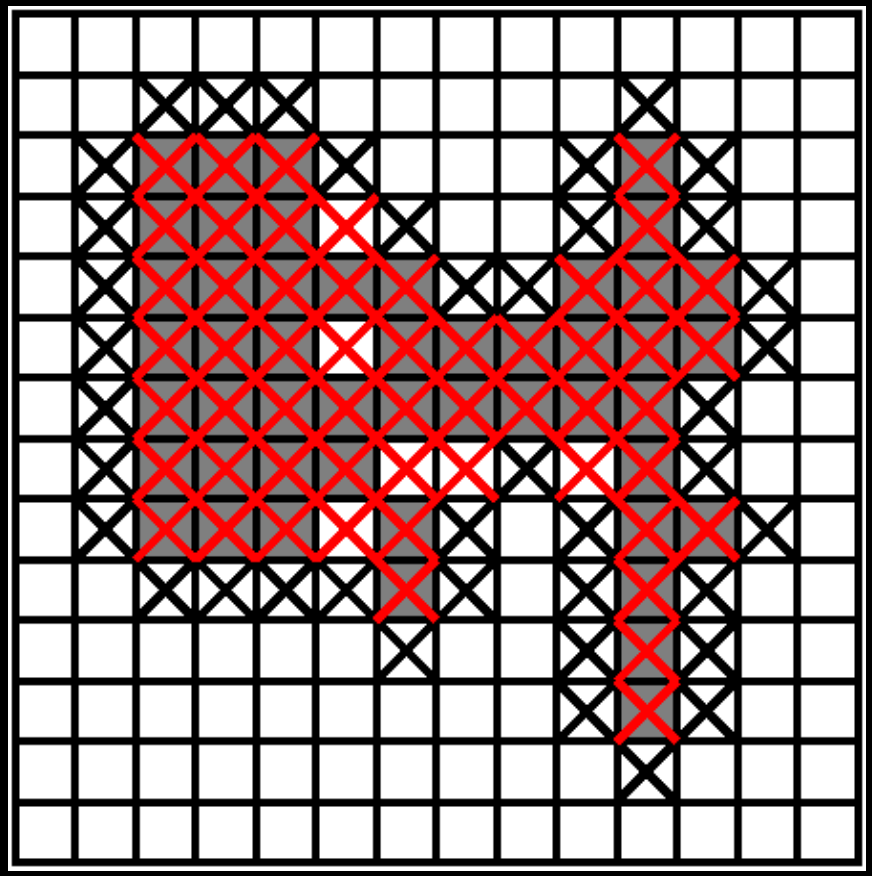
$A \bullet B = (A \oplus B) \ominus B$

Uzavretie

B=

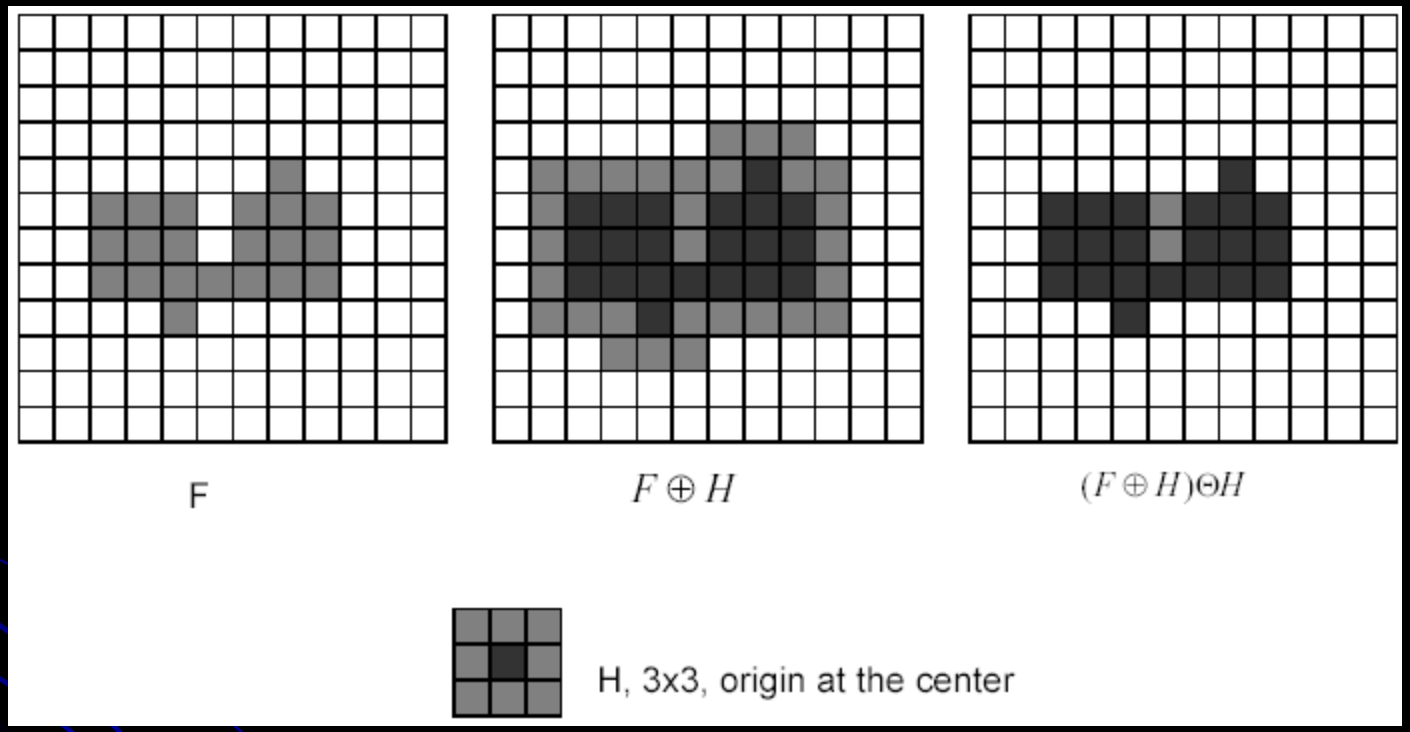


A



A⊕B A • B

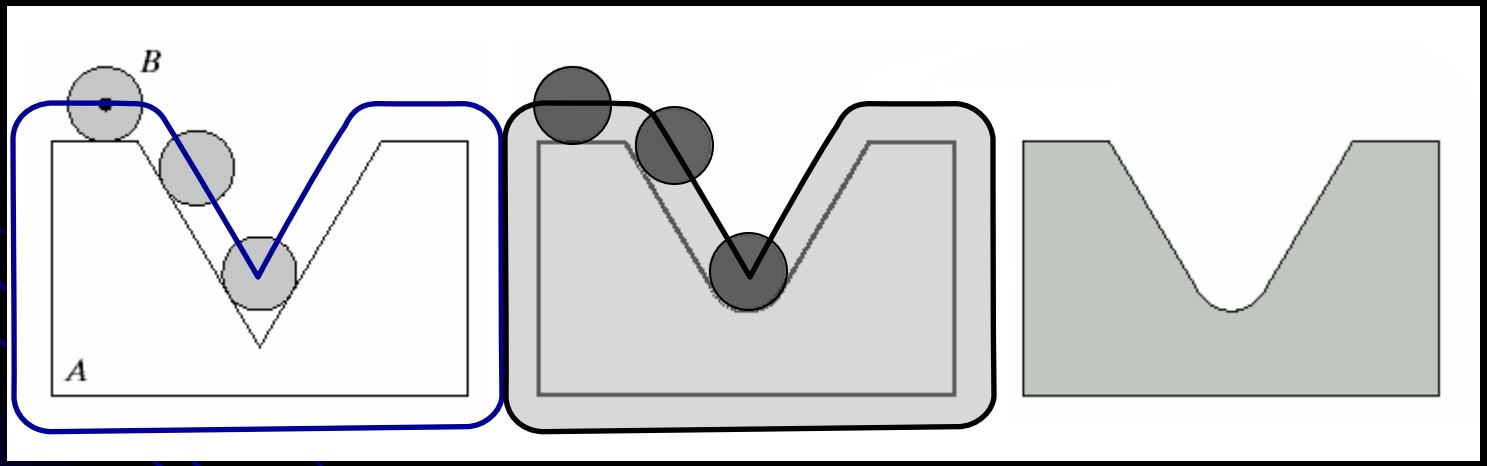
Uzavretie



Uzavretie

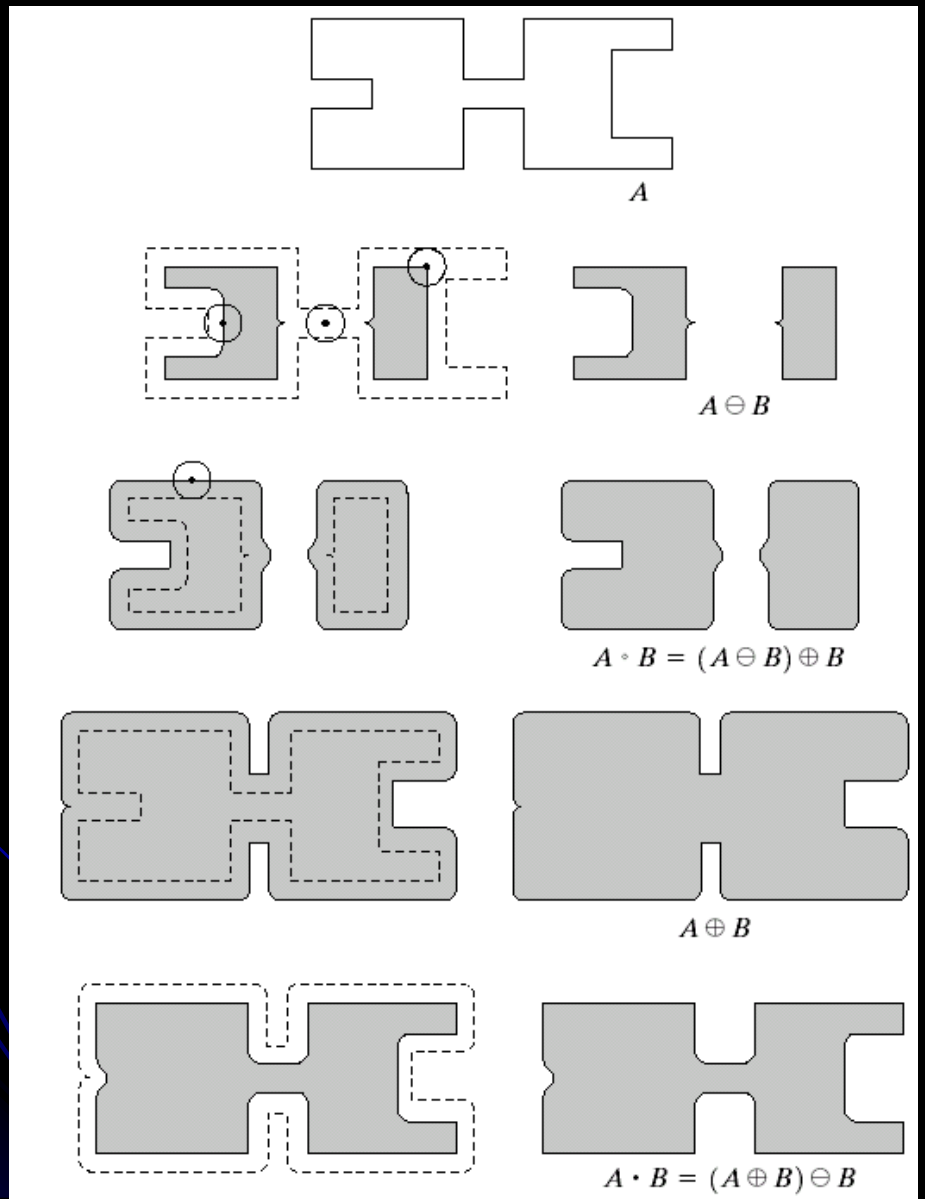
$$A \bullet B = \{w \mid w \in B_x, B_x \cap A \neq \varnothing\}$$

posúvame B **po vonkajšej strane hranice** A

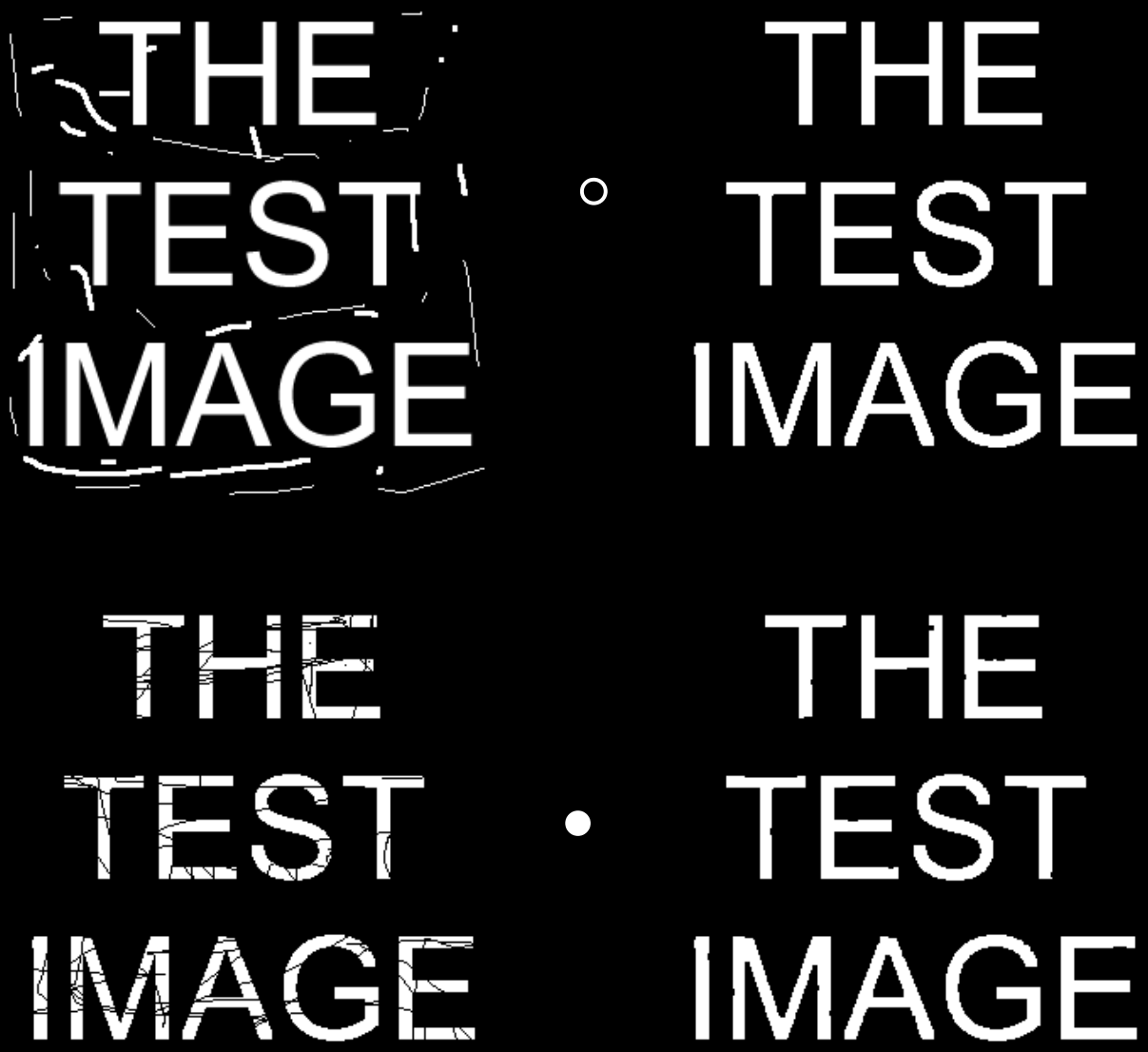


$A \oplus B$ $\ominus B$ $A \bullet B$

Otvorenie - Uzavretie



Otvorenie - Uzavretie



Dualita

$$\left(A \bullet E\right)^{C} = A^{C} \,\mathrm{o}\hat{E}$$

Aplikácia: filtrovanie šumu

1. erózia
$A \ominus B$

2. dilatácia
$(A \ominus B) \oplus B =$
$A \circ B$

3. dilatácia
$(A \circ B) \oplus B$

4. erózia
$((A \circ B) \oplus B) \ominus B =$
$(A \circ B) \bullet B$

Opakovanie

otvorenie

erózia + dilatácia

- vyhladzuje kontúry
- prerušuje tenké spojenia
- maže tenké výčnelky

uzavretie

dilatácia + erózia

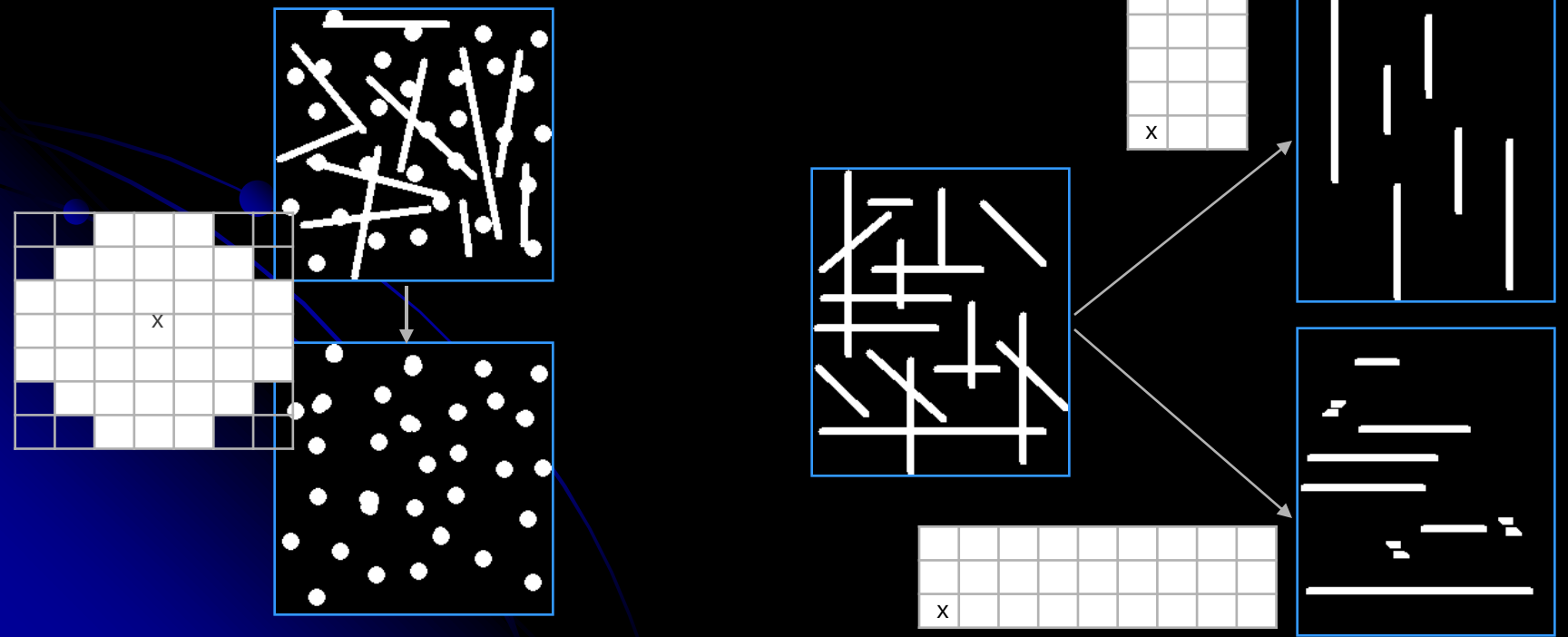
- vyhladzuje kontúry
- spája blízke oblasti
- vypĺňa malé diery a tenké zálivy

Zachovávajú (približnú) veľkosť množiny

Detekcia tvarov

Otvorenie použitím daného štrukturálneho prvku

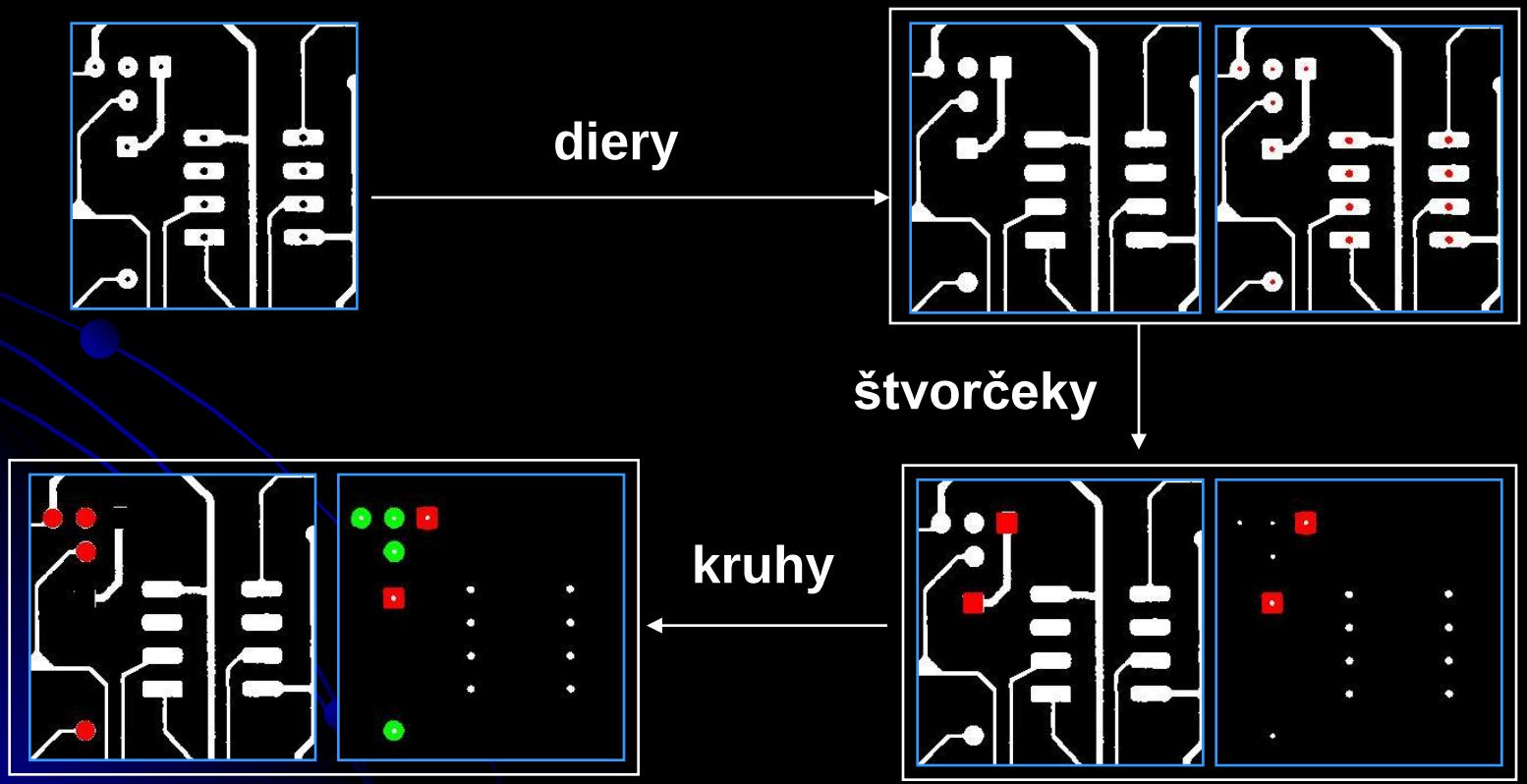
Príklad 1: detekcia kruhov, čiar



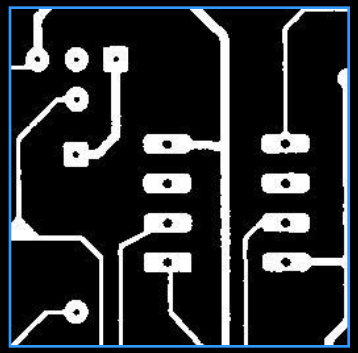
Detekcia tvarov

Príklad 2:

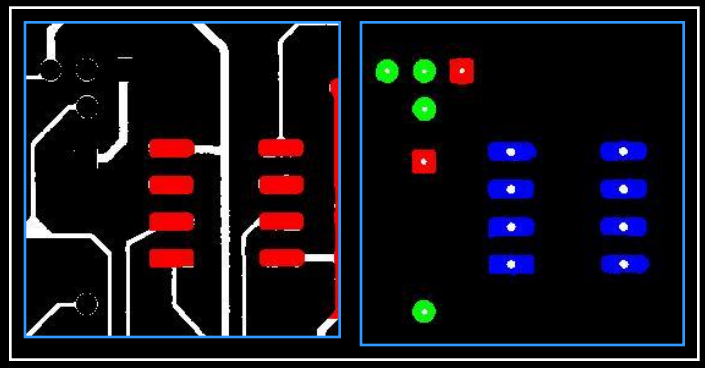
detekcia jednotlivých prvkov plošného spoja

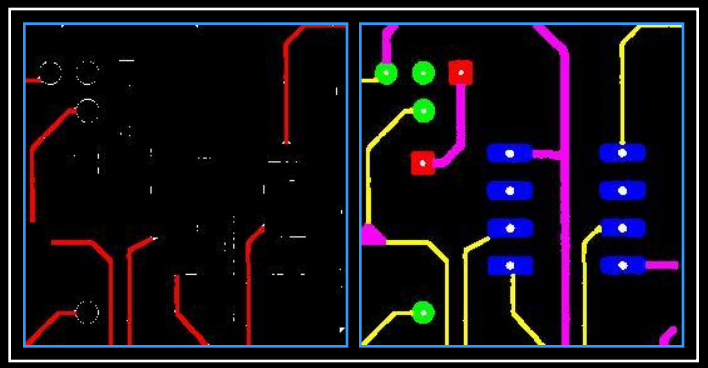


Detekcia tvarov

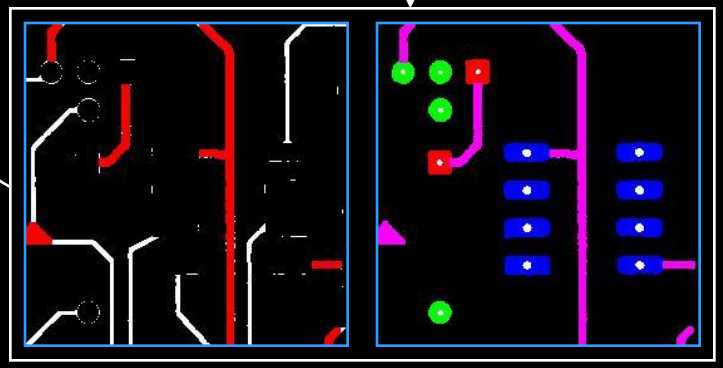


obdĺžniky

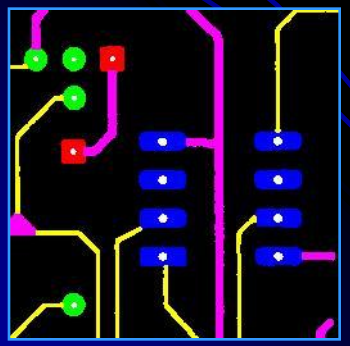




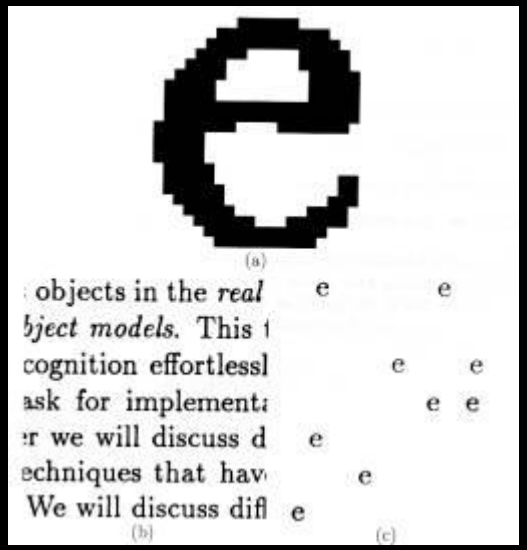
hrubé spoje



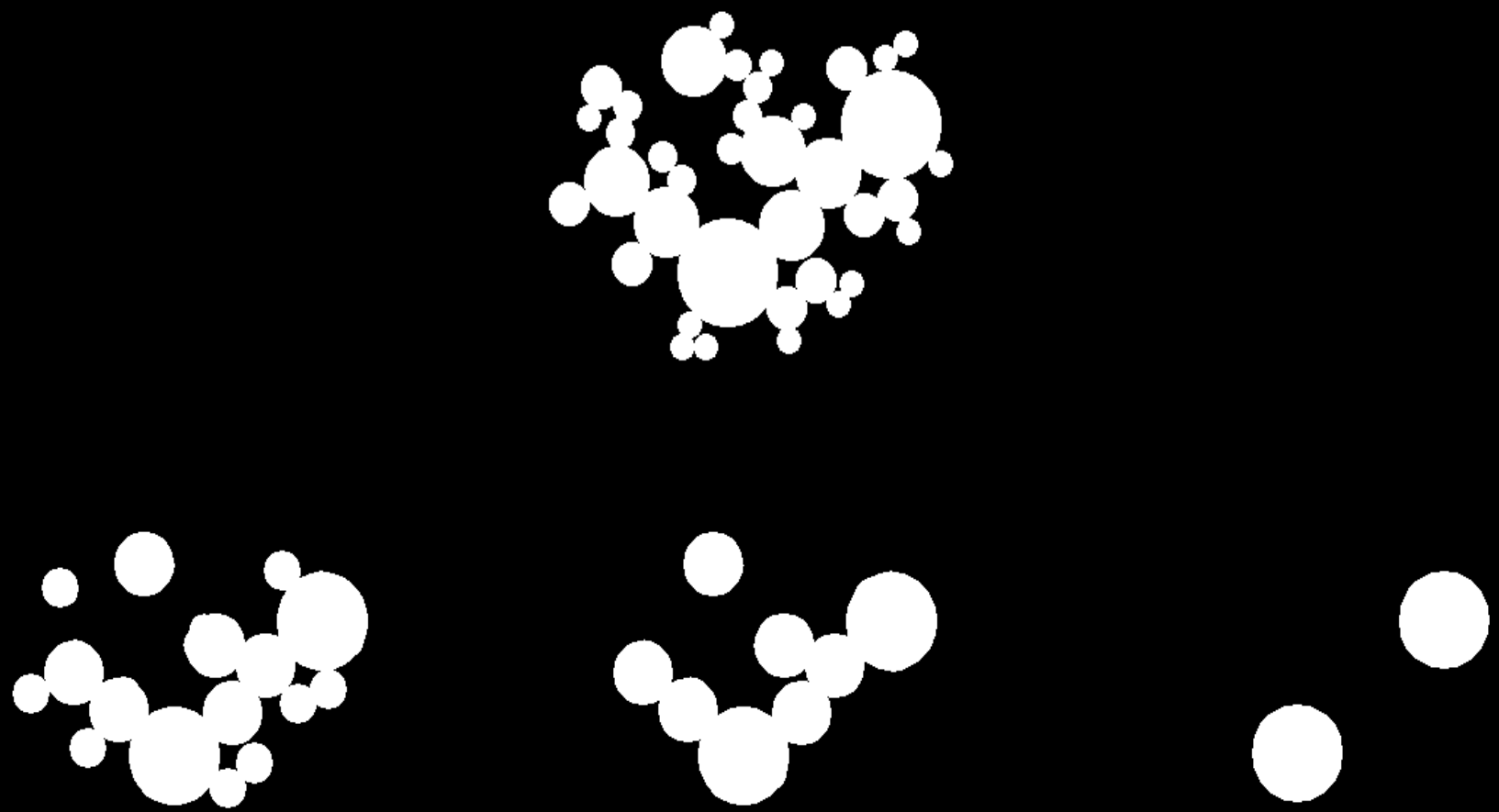
tenké spoje



Text


(a)
objects in the *real*
bject models. This
cognition effortlessl
ask for implementa
r we will discuss d
echniques that hav
We will discuss dif
(b)
e
e
e
e
e
e
e
e
e
(c)

Granulometria

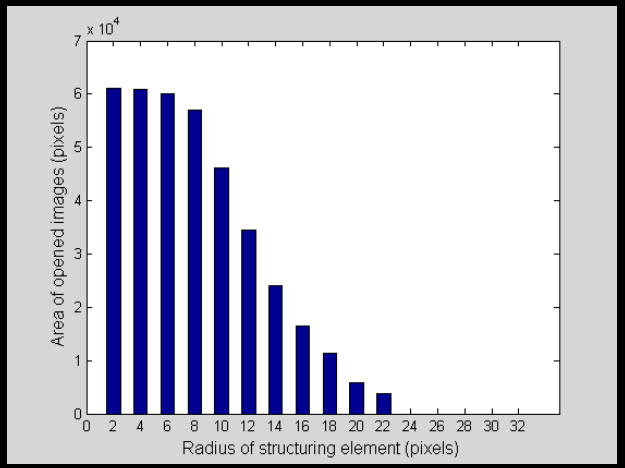


Otvorenie – kruh s priemerom 10, 15 a 25

Granulometria

Otvorenie so zväčšujúcim sa SE:

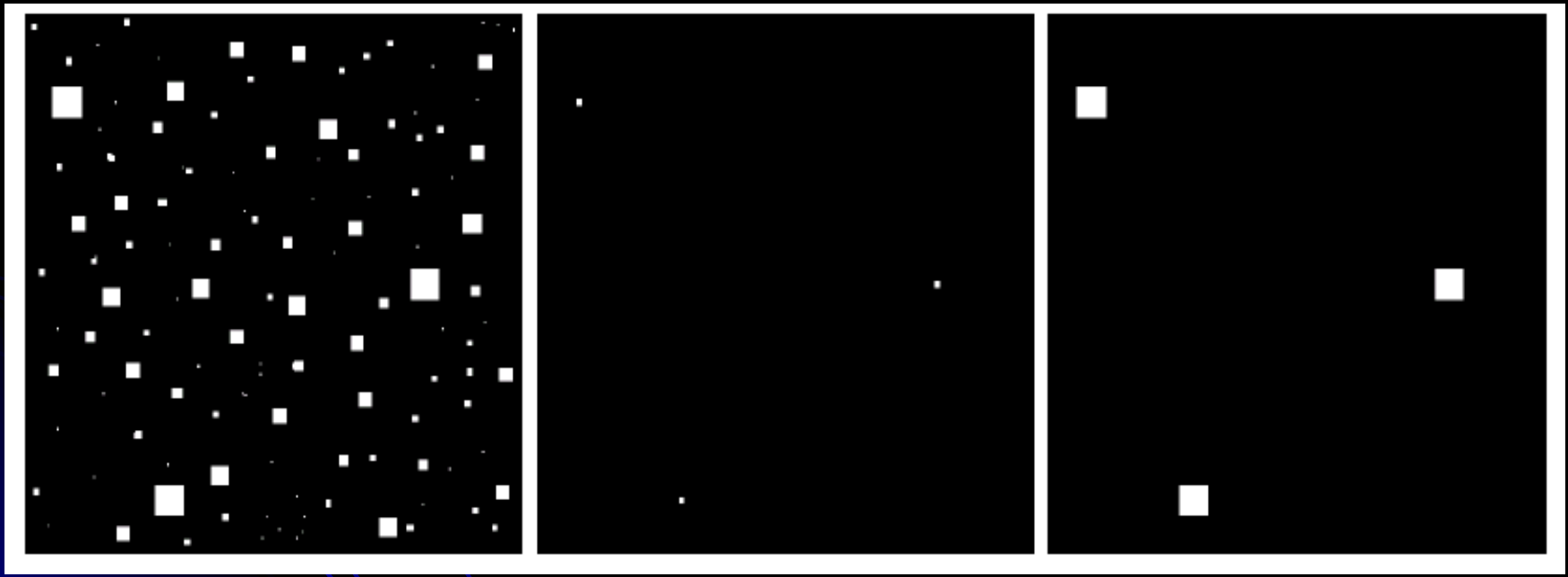
Distribúcia veľkosti granúl →



vstup:
štvorce veľkosti
1x1, 3x3, 5x5, 7x7,
9x9 a 15x15 pixlov

erózia:
ŠP 13x13

dilatácia:
ŠP 13x13



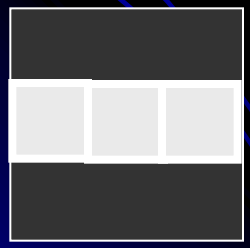
Hit-and-Miss

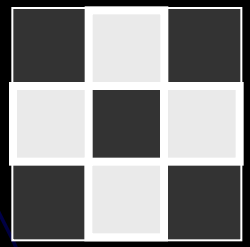
detektor tvarov

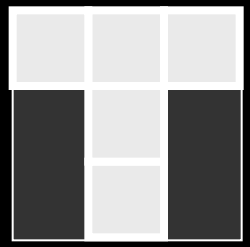
$$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2),$$
$$B_1 \cap B_2 = \varnothing,\ B_1 \cup B_2 = B$$

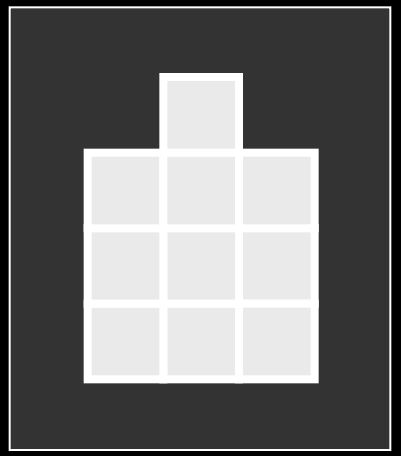
$$A \otimes B = \left(A \ominus B_1\right) - \left(A \oplus \hat{B}_2\right)$$

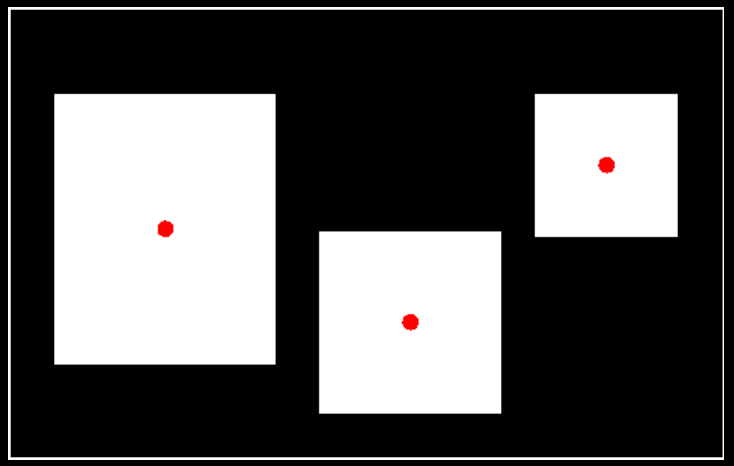
”template matching”



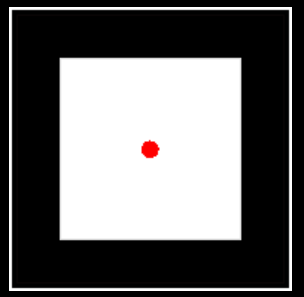




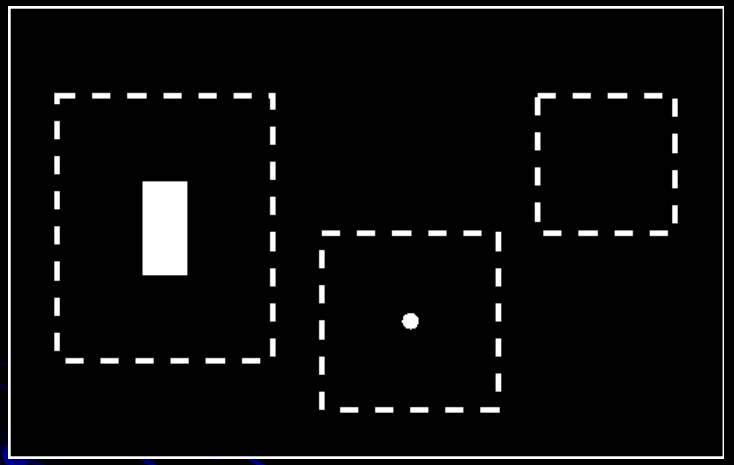




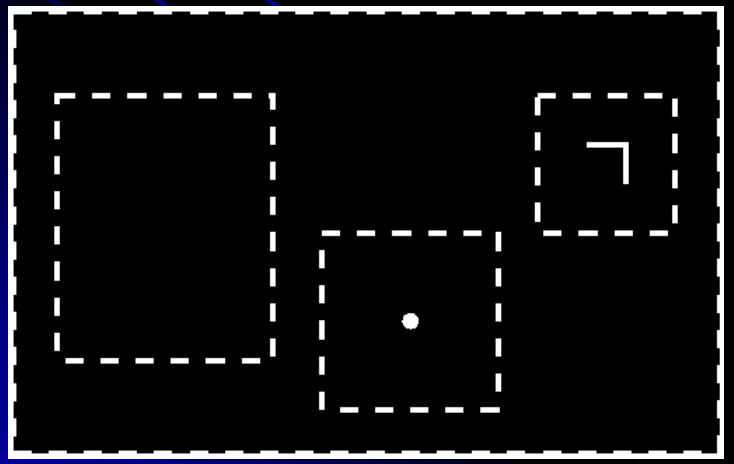
$A=X \cup Y \cup Z$



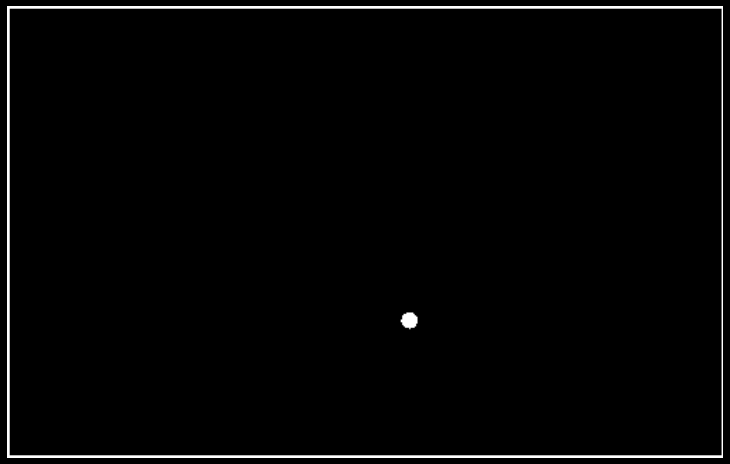
$B_1 \cup B_2$



$A \ominus B_1$

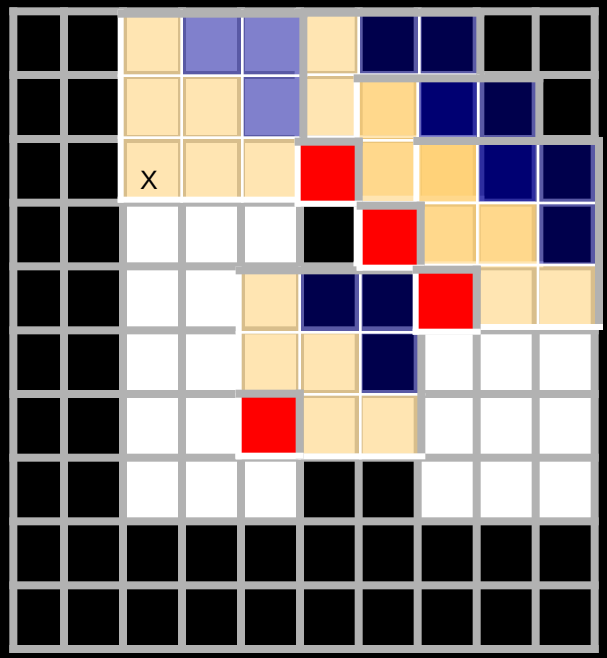


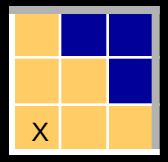
$A^c \ominus B_2$



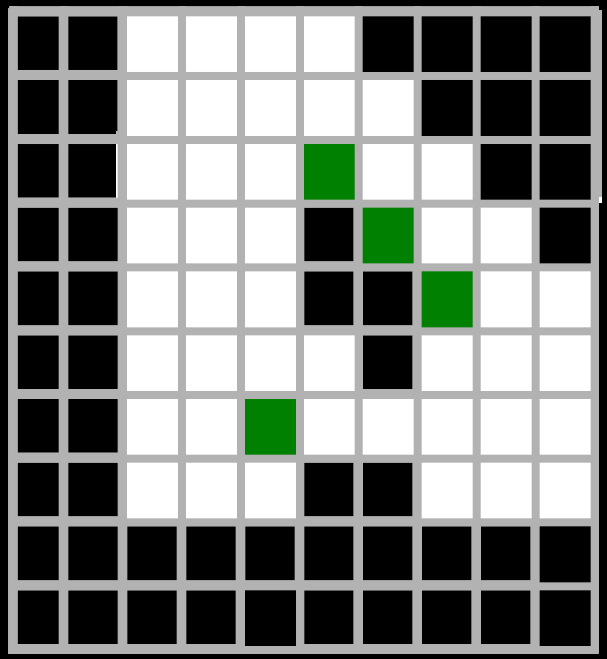
$A \otimes X=(A \ominus B_1) \cap (A^c \ominus B_2)$

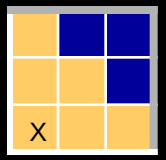
Hit-and-Miss



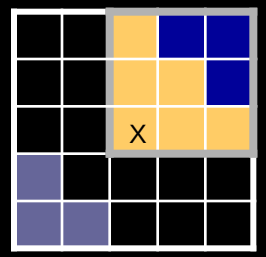


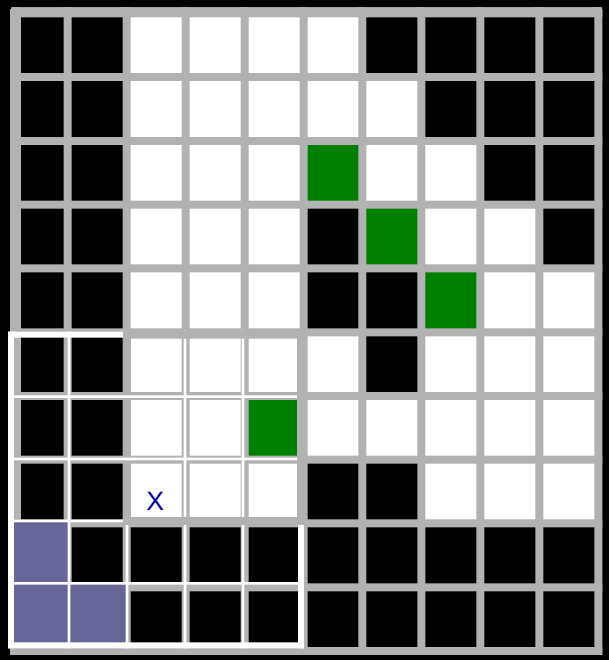
Hit-and-Miss





Hit-and-Miss

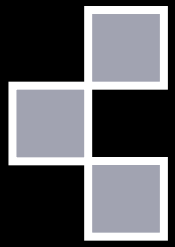




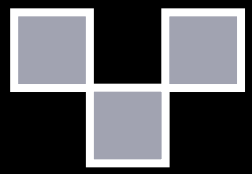
Hit-and-Miss

koncové body

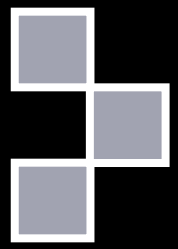
B_1



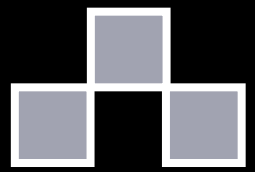
B_2



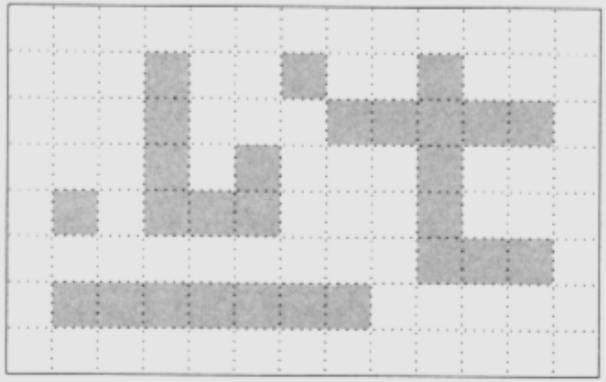
B_2



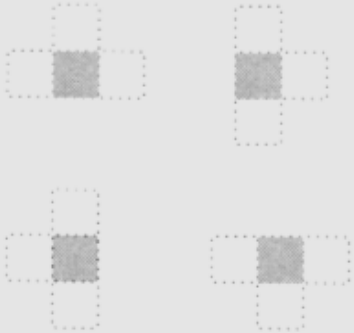
B_2



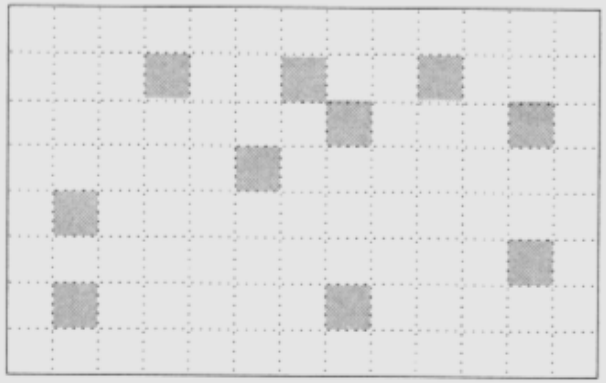
B_2



(a) Input image



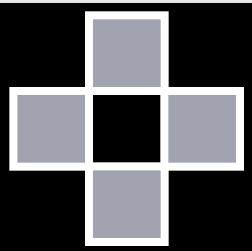
(b) SEs for 4-connected endpoints



(c) Endpoints of input image

izolované body

B_1

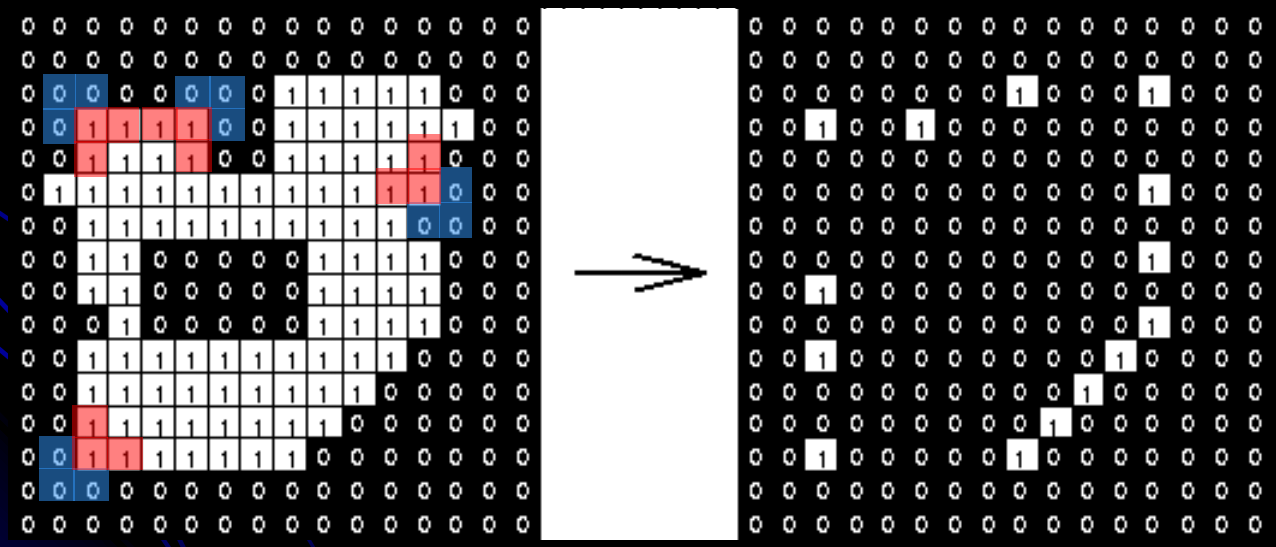


B_2

Hit-and-Miss

Detekcia rohov

	1				1				0	0		0	0				
0	1	1		1	1	0		1	1	0		0	1	1			
0	0				0	0			1				1				



Konvexné rohy

Opakovanie

$\oplus$ dilatácia

$$A \oplus B = \bigcup_{b \in B} A_b$$

$\ominus$ erózia

$$A \ominus B = \bigcap_{b \in B} A_{-b}$$

$\circ$ otvorenie

$$A \circ B = (A \ominus B) \oplus B$$

$\bullet$ uzavretie

$$A \bullet B = (A \oplus B) \ominus B$$

$\otimes$ hit-and-miss

$$A \otimes B = (A \ominus B_1) \cap (A^C \ominus B_2)$$

Rekurzívna erózia

$$F \ominus^{i} K = \begin{cases} F & if \;\; i = 0 \\ (F \ominus^{i-1} K) \ominus K & if \;\; i \geq 1 \end{cases}$$

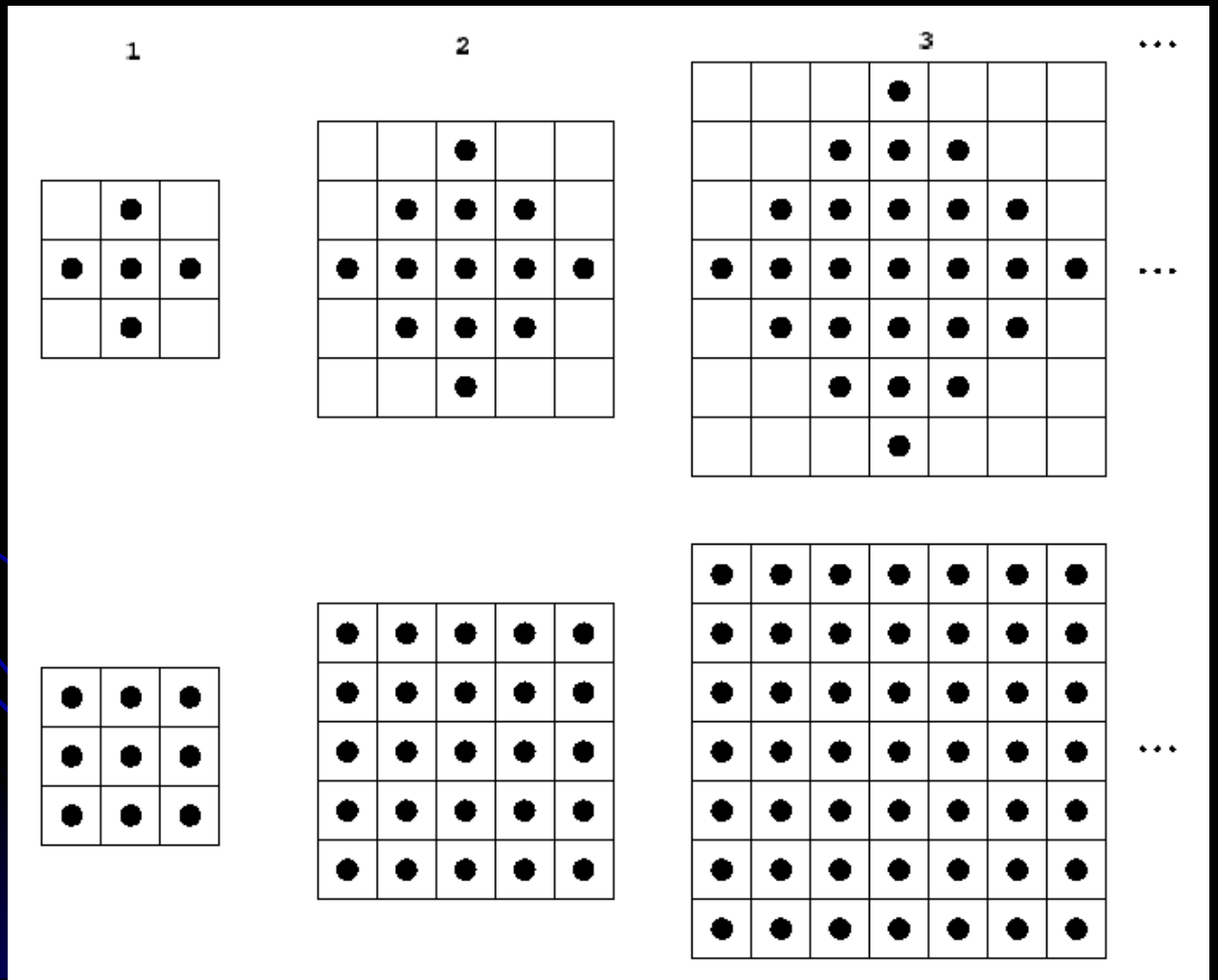
Postupná aplikácia K – množina sa zmenšuje....
až zmizne

$$(A \ominus B) \ominus C = A \ominus (B \oplus C)$$

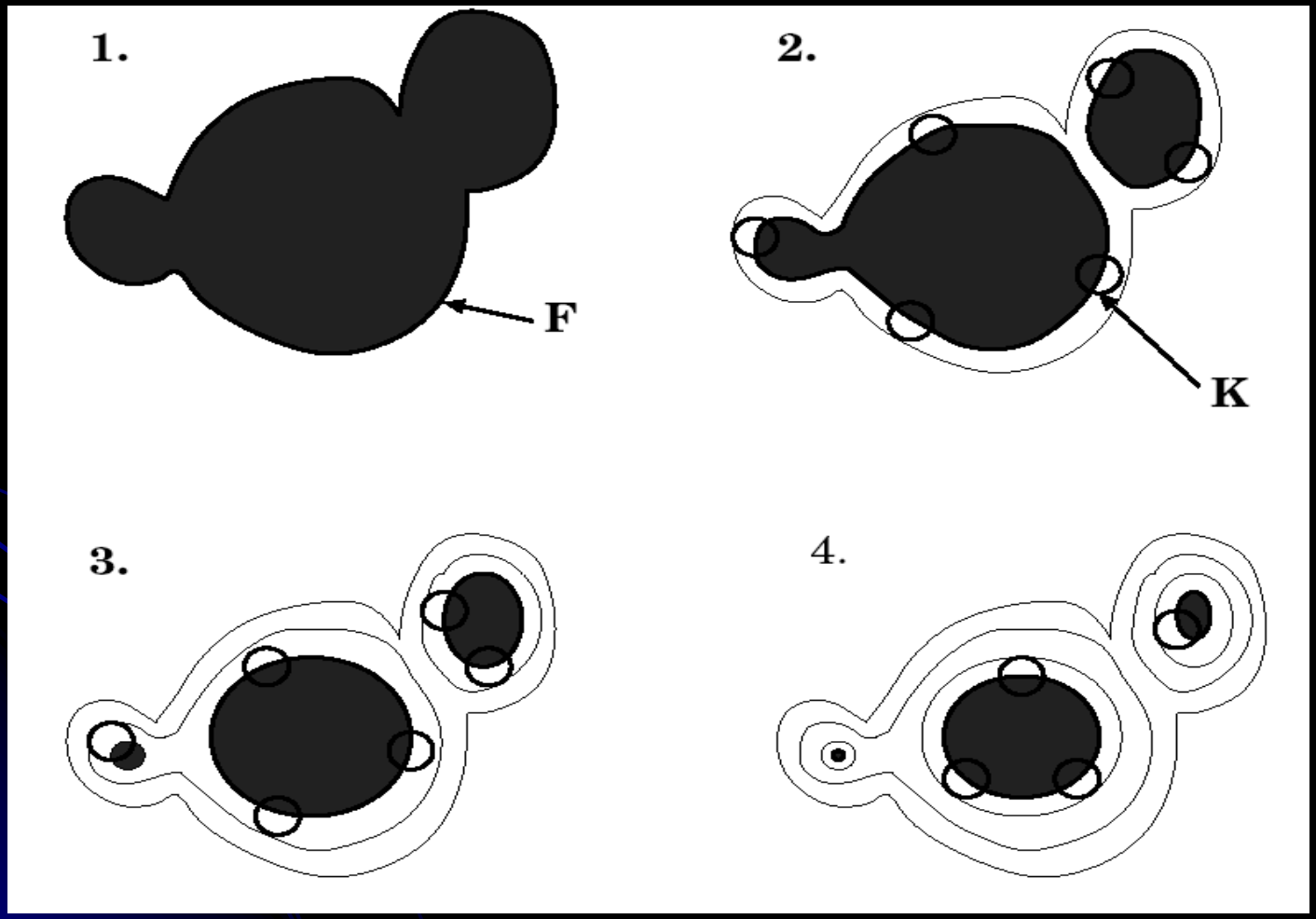
erózia – zväčšujúci sa ŠP „rovnakého“ tvaru

ŠP

B B ⊕ B B ⊕ B ⊕ B



Rekurzívna erózia

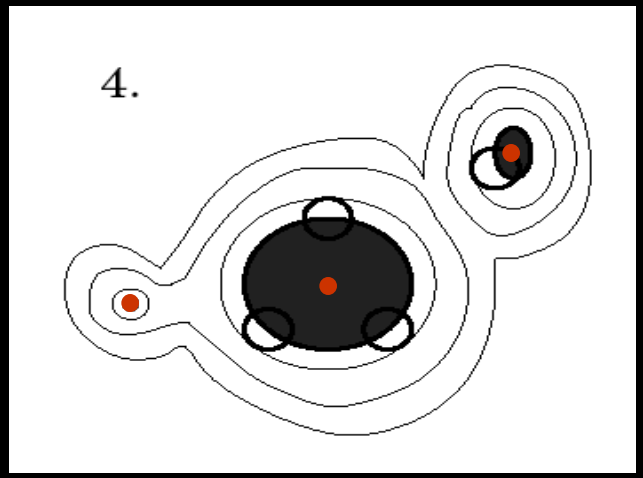


Ultimatívna erózia

Ultimate Erosion (UE) =

Rekurzívna erózia

Zachováme objekty tesne pred zmiznutím



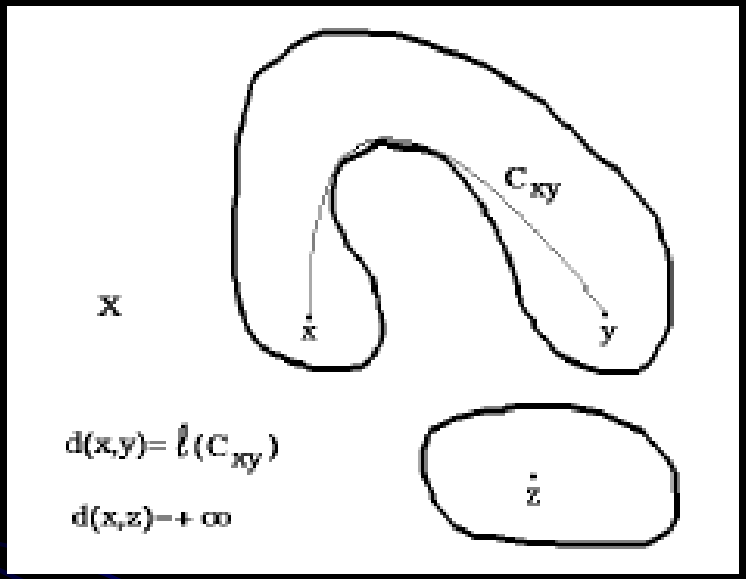
Rekurzívna dilatácia

$$F \overset{i}{\oplus} K = \begin{cases} F & if \;\; i = 0 \\ (F \overset{i-1}{\oplus} K) \oplus K & if \;\; i \geq 1 \end{cases}$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

dilatácia – zväčšujúci sa ŠP „rovnakého“ tvaru

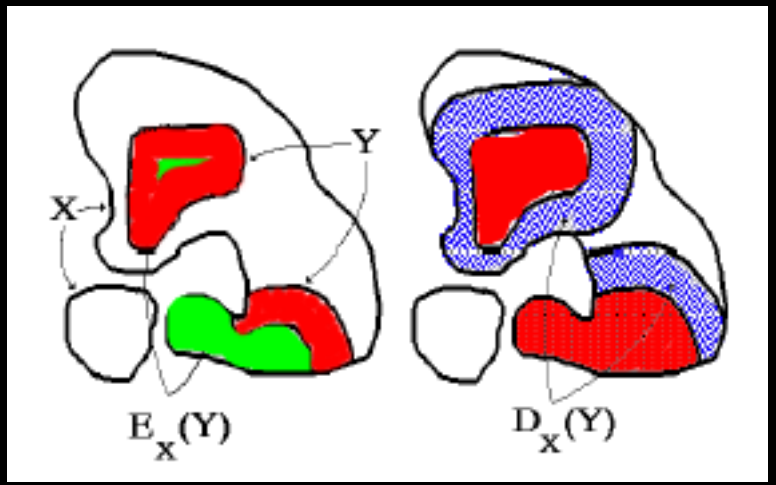
Geodetické transformácie



geodetická vzdielenosť
$d_X(x,y)$

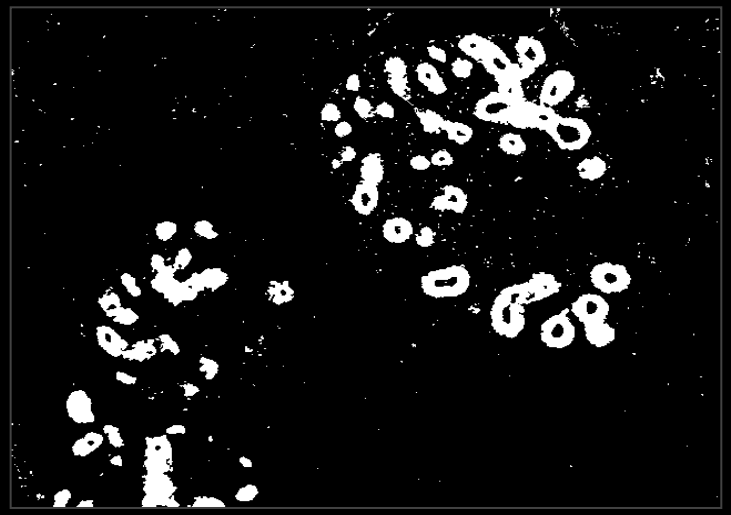
geodetická (podmienená)
dilatácia
erózia

$$Y \oplus|_X B = (Y \oplus B) \cap X$$
$$Y \ominus|_X B = (Y \ominus B) \cap X$$



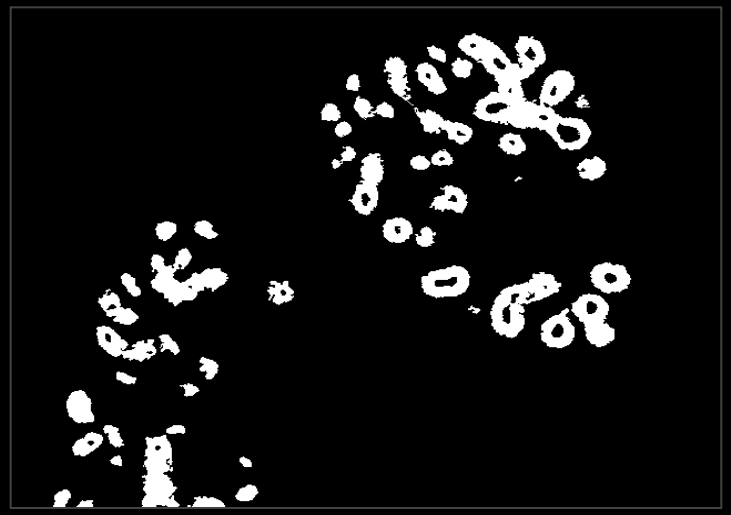
Podmienená dilatácia

C

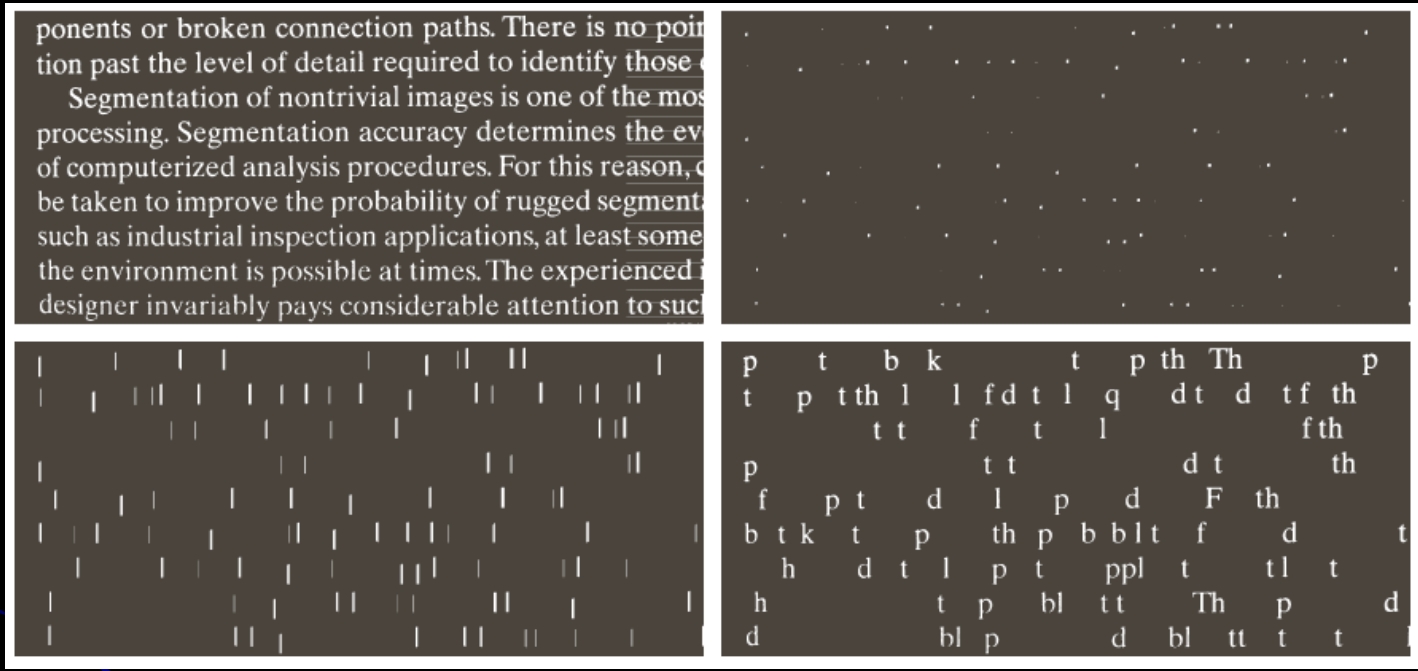


A

$(A \oplus B) \cap C$



Podmienená dilatácia



a b
c d

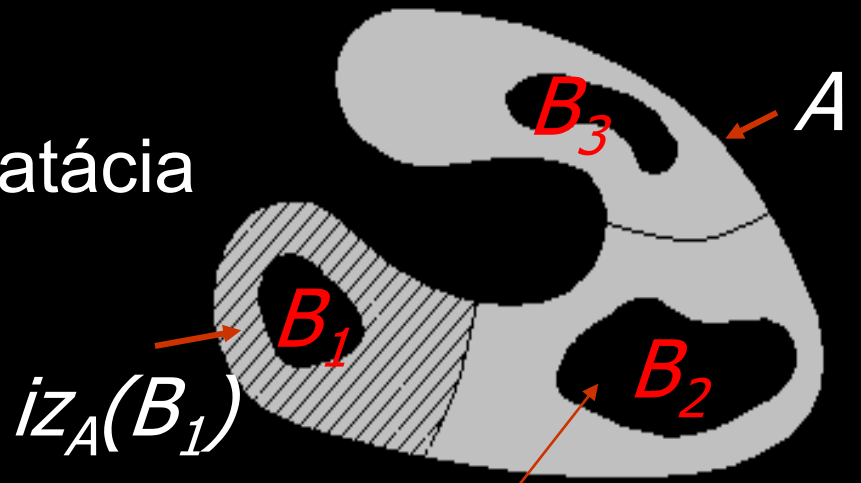
FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

Geodetický vplyv

Geodesic Influence (GI) =

Rekurzívna podmienená dilatácia

– rozdelí množinu



Zóny vplyvu (influence zones) zárodok

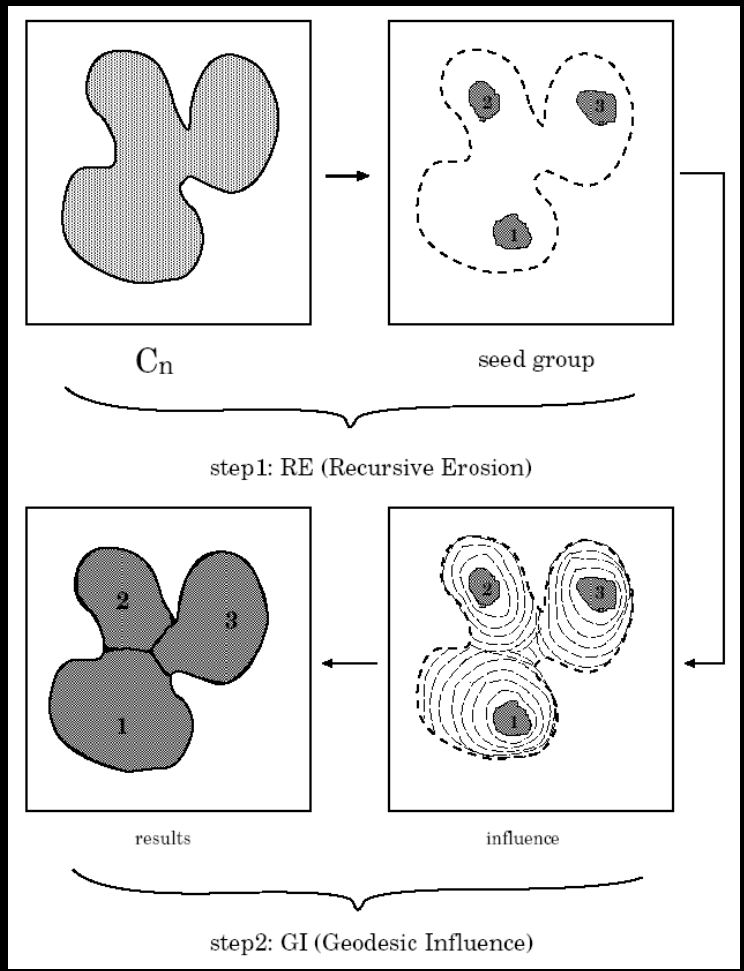
$$iz_A(B_i) = \{p \in A, \forall_{j \neq i}\, d_A(p, B_i) < d_A(p, B_j)\}$$

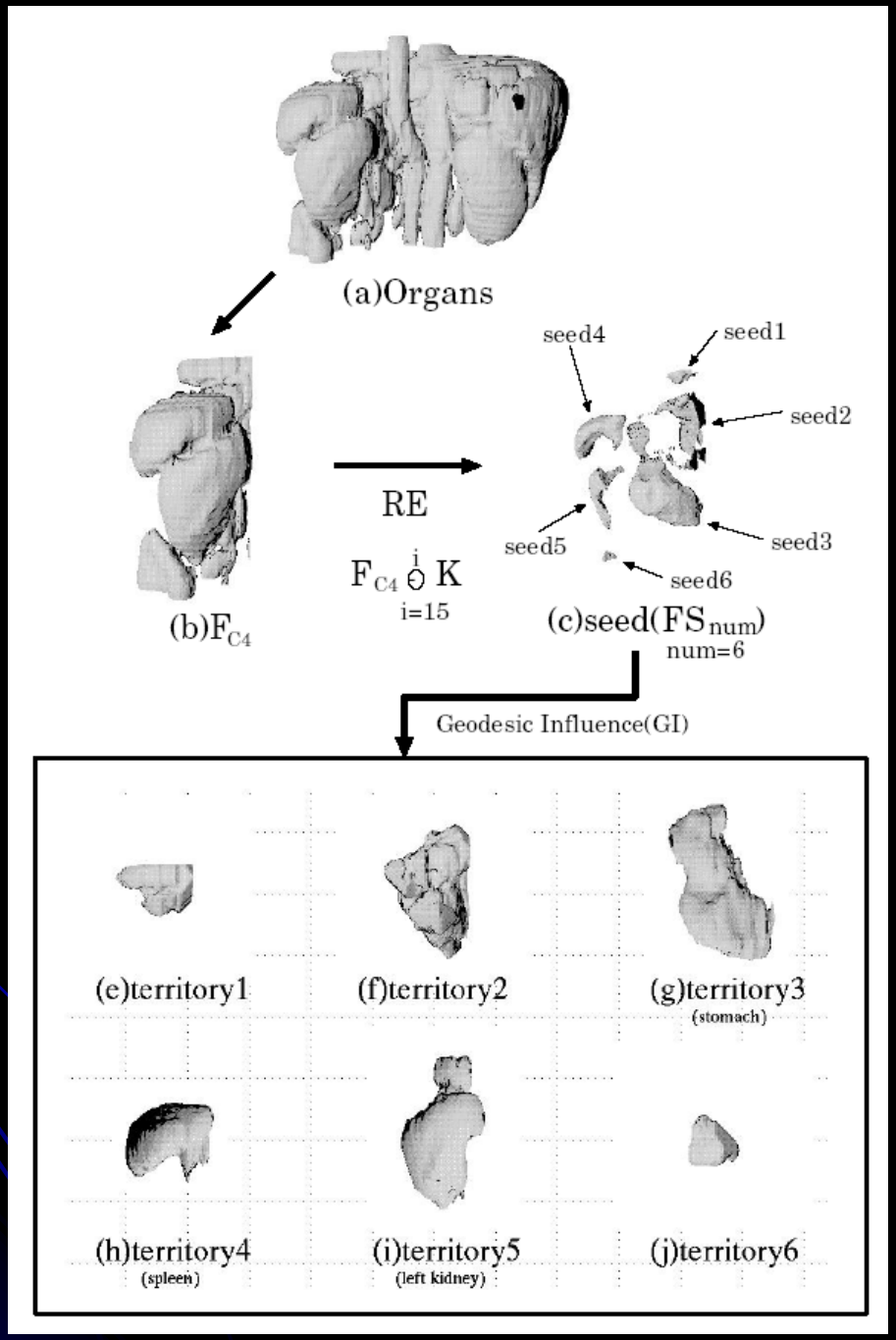
Voronoiov diagram

UE a GI

UE: rozdelí región, zachová zárodky

GI: zo zárodkov rekonštruuje objekt

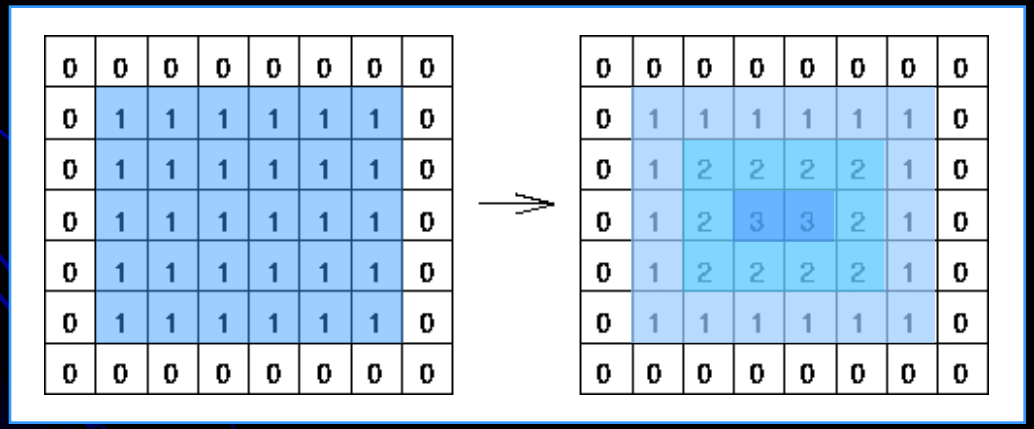



(a)Organs
(b)F_{C4}
RE
$F_{C4} \ominus^{i} K$
i=15
seed4
seed1
seed2
seed5
seed3
seed6
(c)seed(FS_{num})
num=6
Geodesic Influence(GI)
(e)territory1
(f)territory2
(g)territory3
(stomach)
(h)territory4
(spleen)
(i)territory5
(left kidney)
(j)territory6

Distance Transform

Operátor aplikovaný na binárne obrazy

Úrovne šedej = vzdialenosť od najbližšieho okraja



Distance Transform

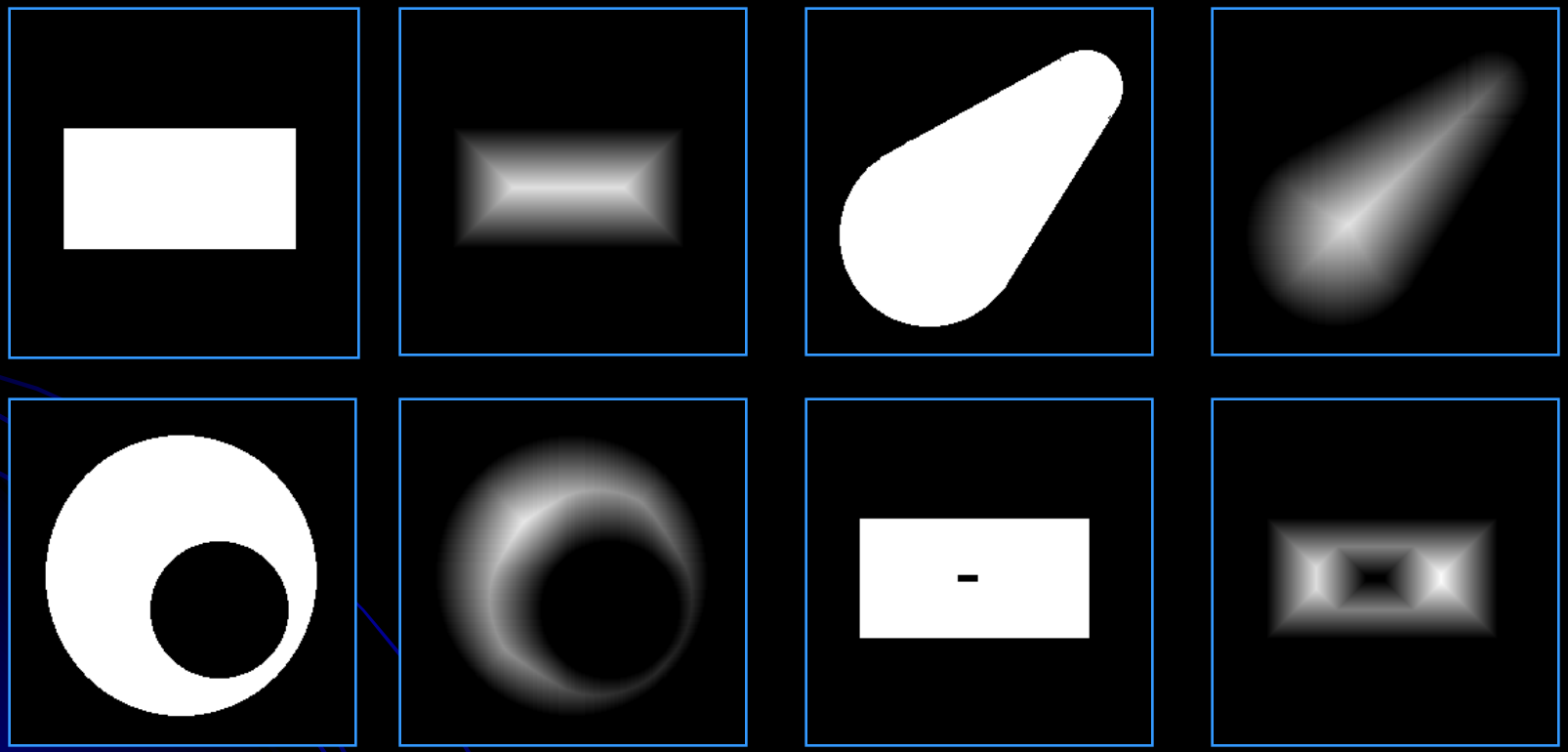
Rekurzívna erózia – kým útvar nezmizne

Vzdialenosť – číslo iterácie pri zmiznutí bodu

Vhodné ŠP:

- štvorcový – chessboard
- krížový – city block
- kruhový – Euklidovská

Distance Transform



Hľadanie hranice

dilatácia – erózia

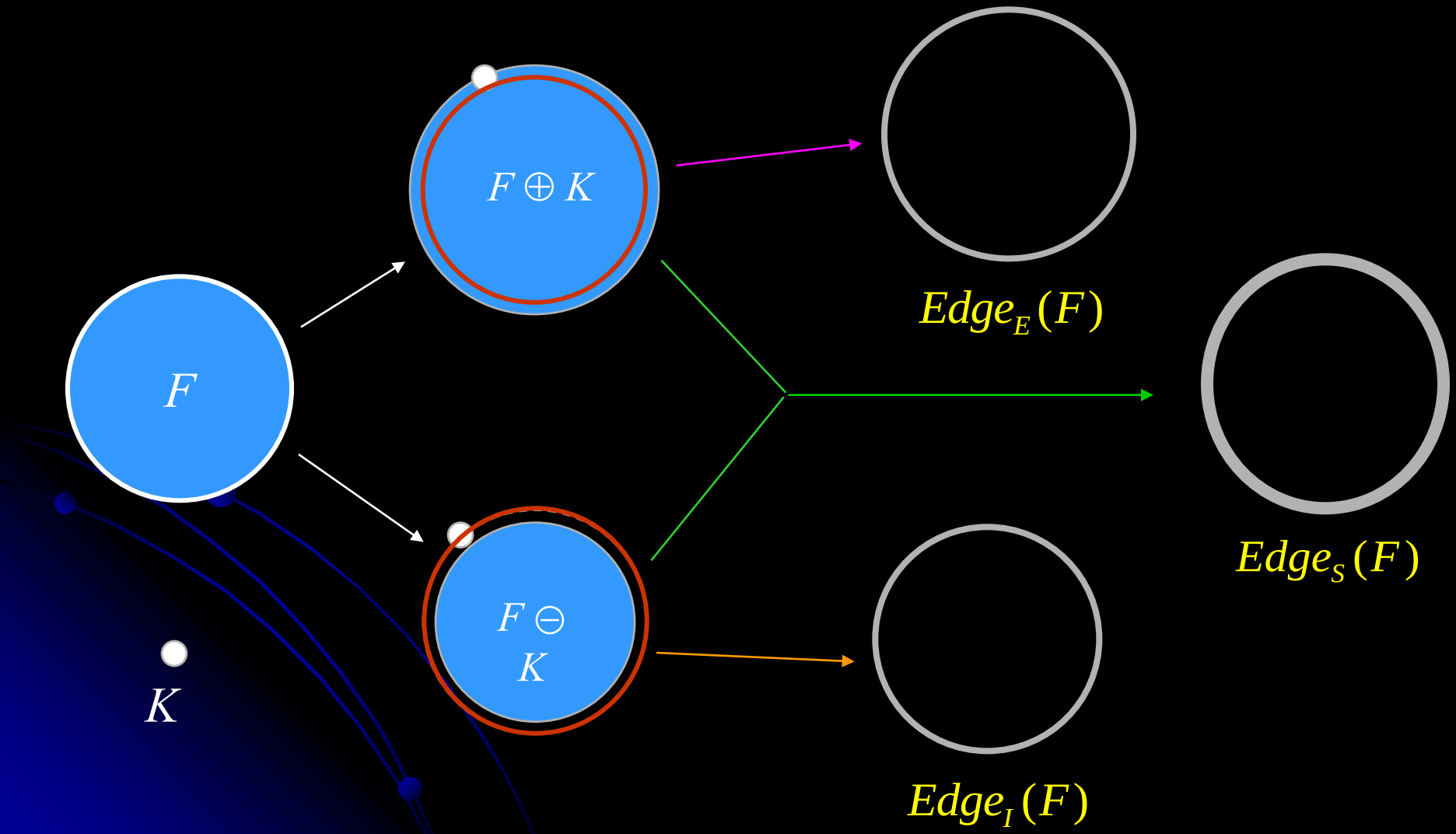
Algoritmy:

Štandard: $Edge_S(F) = (F \oplus K) - (F \ominus K)$

Externé: $Edge_E(F) = (F \oplus K) - F$

Interné: $Edge_I(F) = F - (F \ominus K)$

Hľadanie hranice



Hľadanie hranice

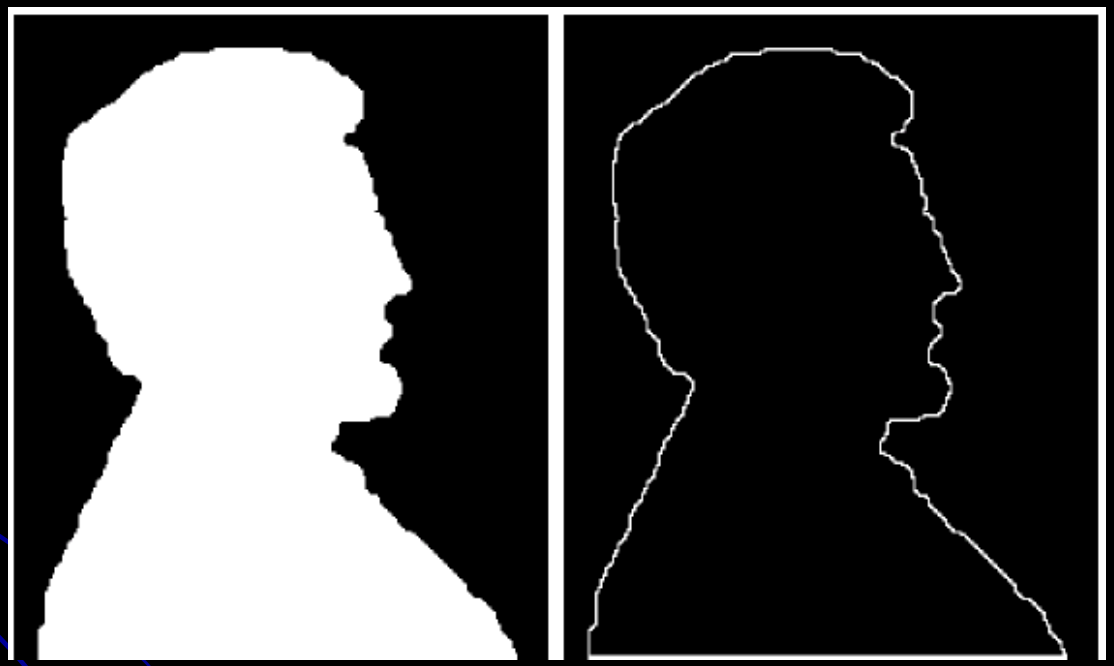
F

$Edge_S(F)$

$Edge_E(F)$

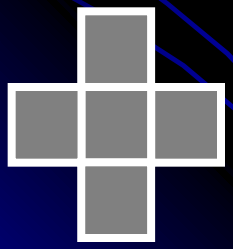
$Edge_I(F)$

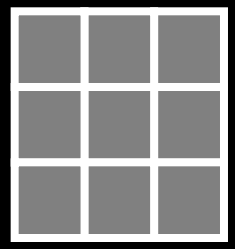
Hľadanie hranice



Hľadanie hranice

$$\beta(A) = A - (A \ominus B)$$





8-súvislá hranica

4-súvislá hranica

Opakovanie

otvorenie

erózia + dilatácia

- vyhladzuje kontúry
- prerušuje tenké spojenia
- maže tenké výčnelky

uzavretie

dilatácia + erózia

- vyhladzuje kontúry
- spája blízke oblasti
- vypĺňa malé diery a tenké zálivy

Zachovávajú (približnú) veľkosť množiny

Opakovanie

⊕ dilatácia

$$A \oplus B = \bigcup_{b \in B} A_b$$

⊖ erózia

$$A \ominus B = \bigcap_{b \in B} A_{-b}$$

∘ otvorenie

$$A \circ B = (A \ominus B) \oplus B$$

• uzavretie

$$A \bullet B = (A \oplus B) \ominus B$$

⊗ hit-and-miss

$$A \otimes B = (A \ominus B_1) \cap (A^C \ominus B_2)$$

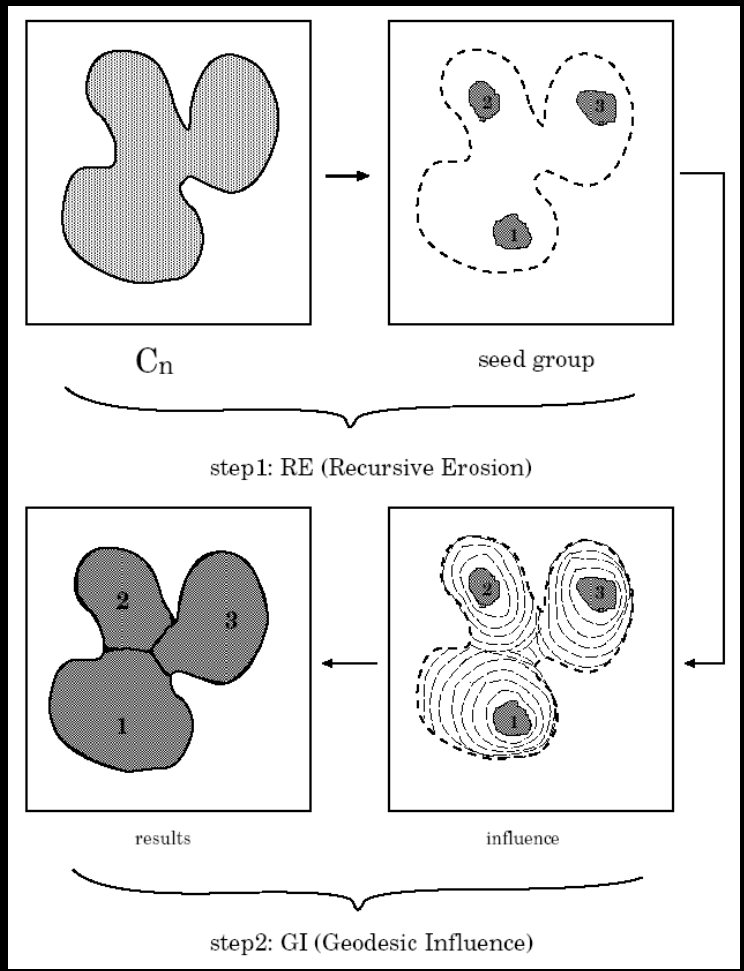
UE a GI

Ultimatívna erózia (UE):
rozdelí región, zachová zárodky

Geodesic Influence (GI)

Rekurzívna podmienená dilatácia

: zo zárodkov rekonštruuje objekt



Hľadanie hranice

dilatácia – erózia

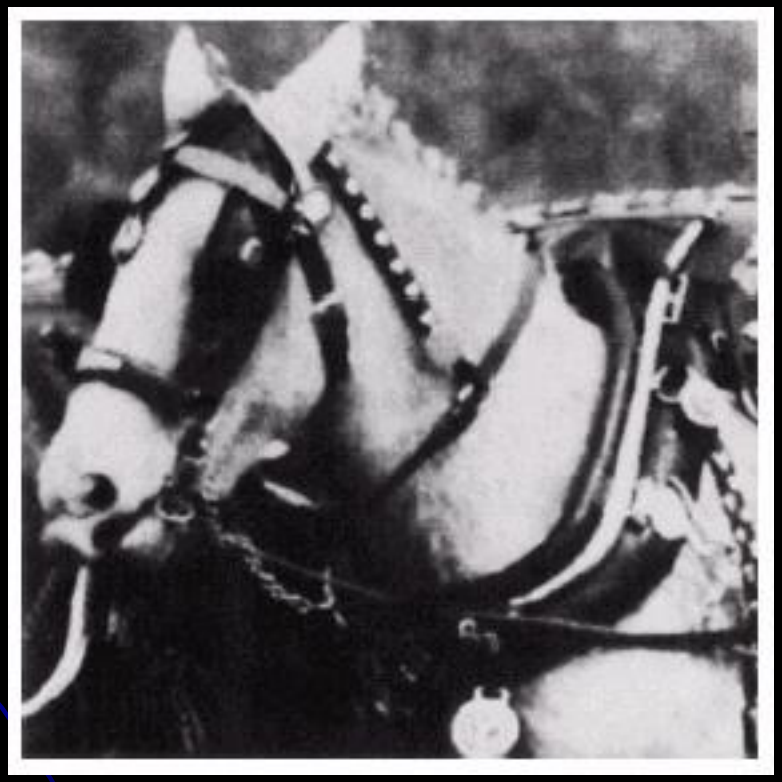
Algoritmy:

Štandard: $Edge_S(F) = (F \oplus K) - (F \ominus K)$

Externé: $Edge_E(F) = (F \oplus K) - F$

Interné: $Edge_I(F) = F - (F \ominus K)$

Šedotónová morfológia



Obraz

$X = \{ (\boldsymbol{a}; f_X(\boldsymbol{a})) \mid \boldsymbol{a} \in E^{n-1}, f_X(\boldsymbol{a}) \in R \cup \{\infty\} \cup \{-\infty\} \}$

= n-dim graf

Nosič (support): $\text{supp}(X) = \{ \boldsymbol{a} \in E^{n-1}, f(\boldsymbol{a}) \in R \}$

mimo: ∞ alebo $-\infty$

Pre nás n=3

Obraz

Binárny obraz
$f(\boldsymbol{x})$ = konštanta

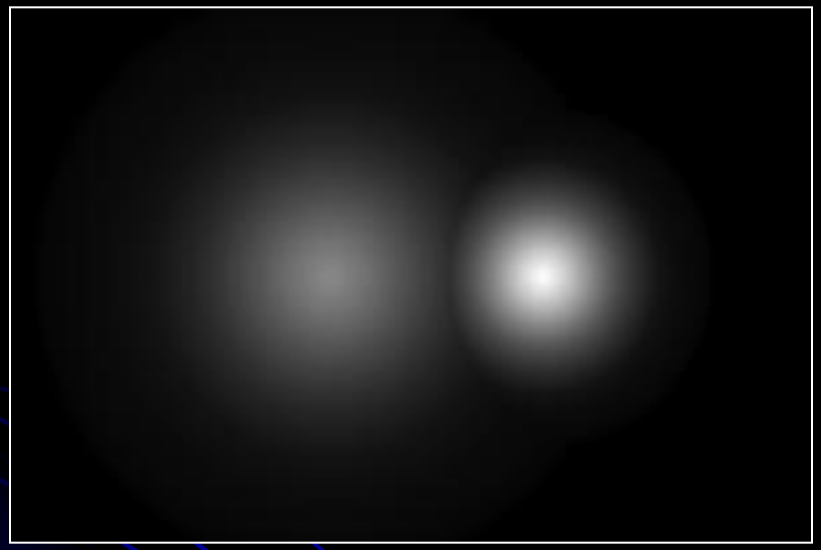
Šedotónový obraz
$f(\boldsymbol{x})$ – úrovne intenzity

$X \subseteq Y$:
$\text{supp}(X) \subseteq \text{supp}(Y)$
$f_X(\boldsymbol{a}) \leq f_Y(\boldsymbol{a})$ pre $\boldsymbol{a} \in \text{supp}(X)$

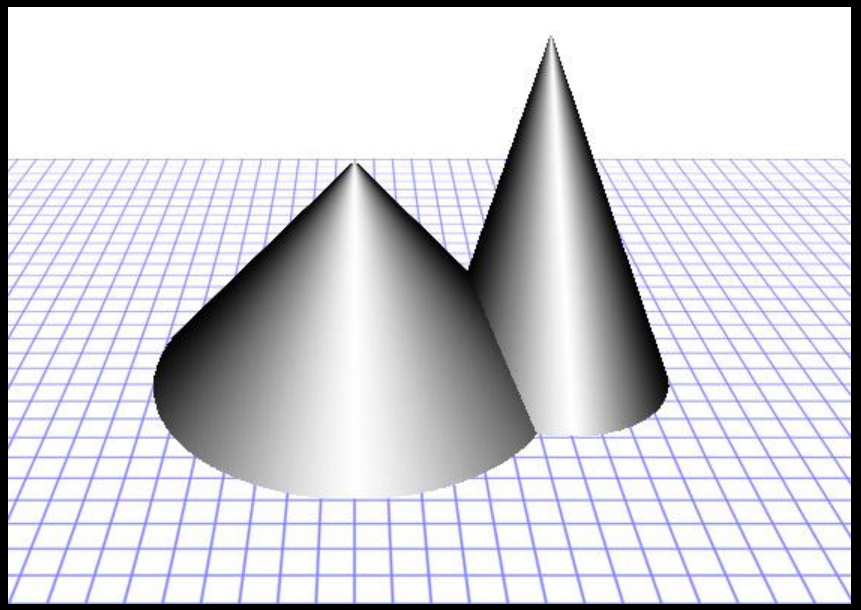
Množinové operácie

min/max
sup/inf

Obraz

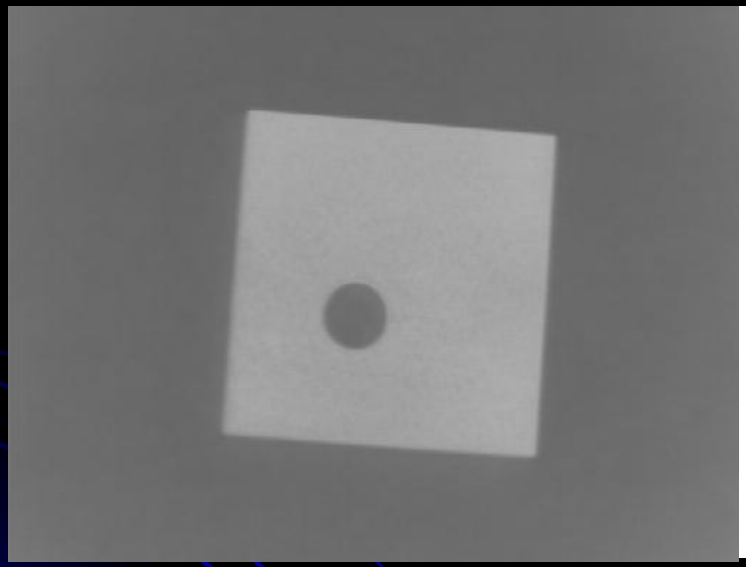


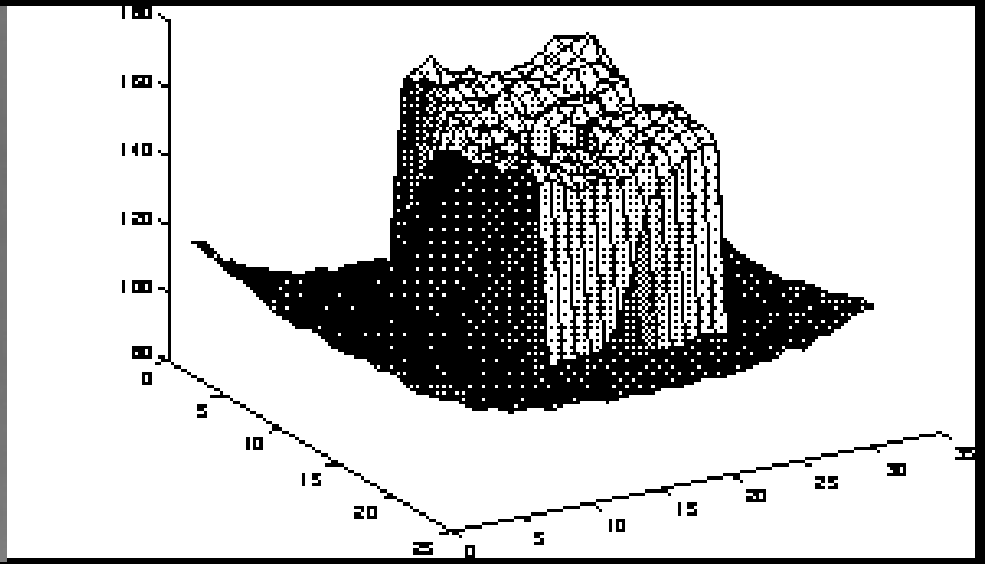
obraz



3D reprezentácia

Obraz

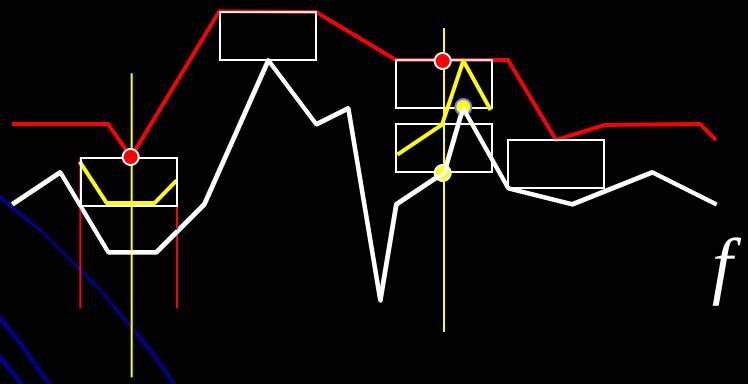


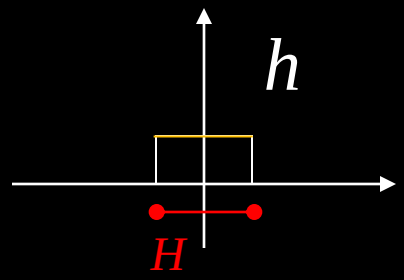


Dilatácia

$$(f \oplus h)(x,y) = \max_{(r,s)\in H} \{f(x-r, y-s) + h(r,s)\}$$

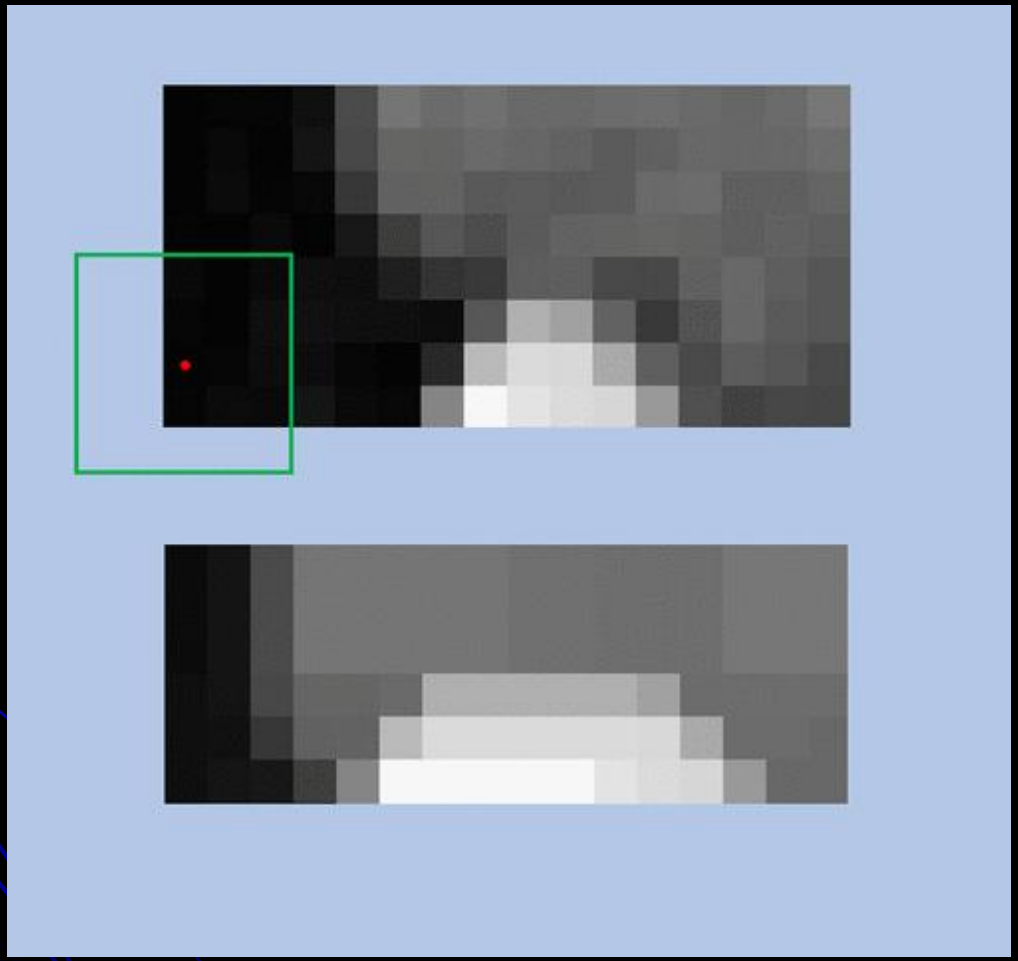
$$(f \oplus h)(x) = \max_{r\in H} \{f(x-r) + h(r)\}$$





H(x) - "structuring functions"

Dilatácia



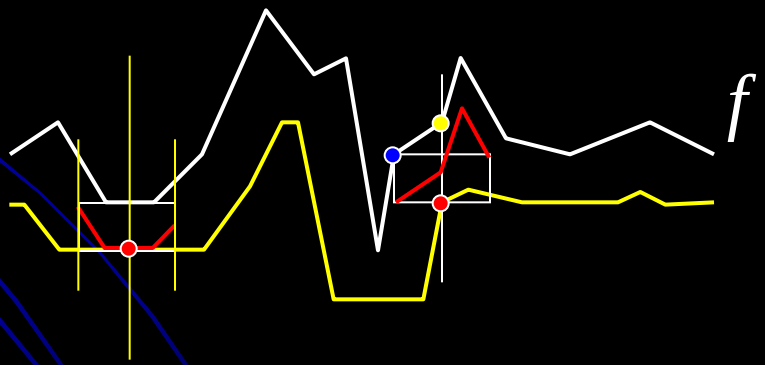
Dilatácia

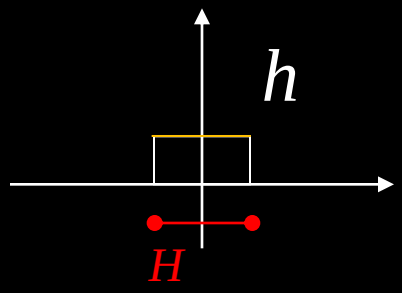
Zjasňuje obrázok – zvyšuje intenzitu

Erózia

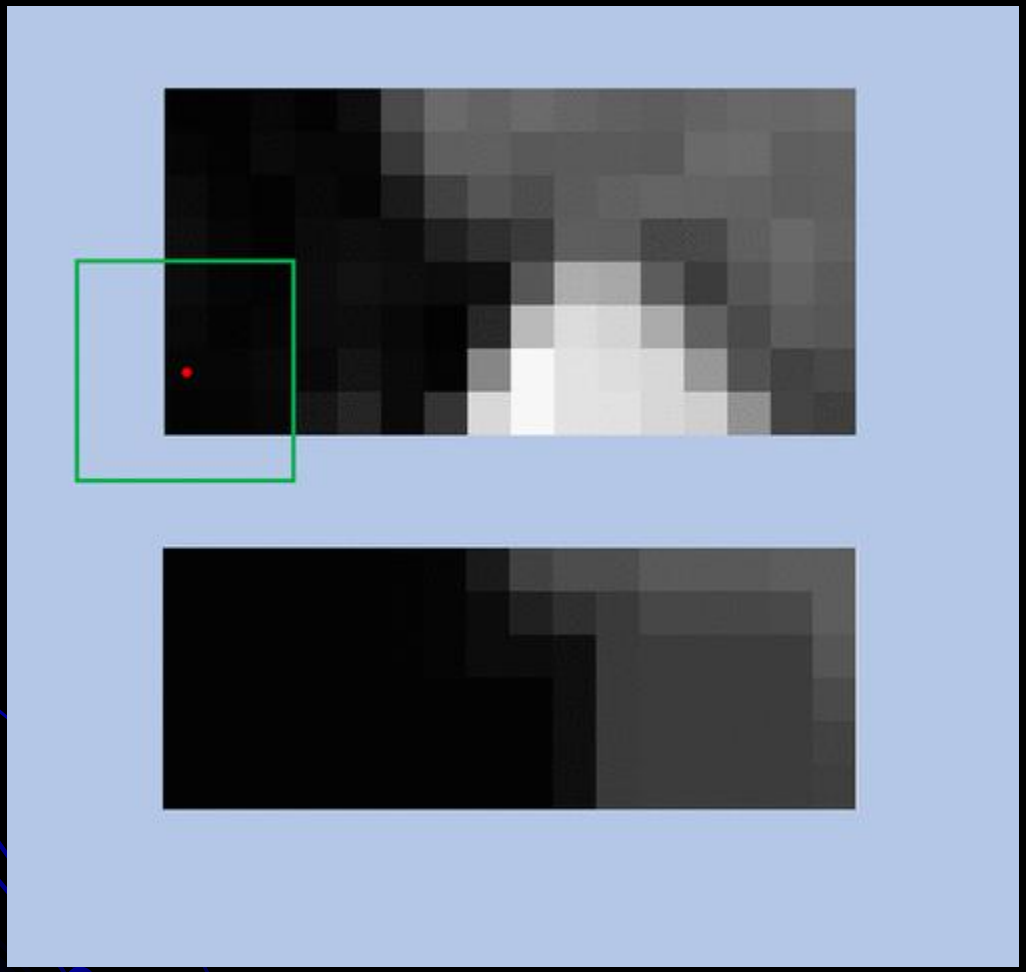
$$(f \ominus h)(x,y) = \min_{(r,s)\in H} \{f(x+r, y+s) - h(r,s)\}$$

$$(f \ominus h)(x) = \min_{r\in H} \{f(x+r) - h(r)\}$$





Erózia



Erózia

Ztmavuje obrázok – znižuje intenzitu

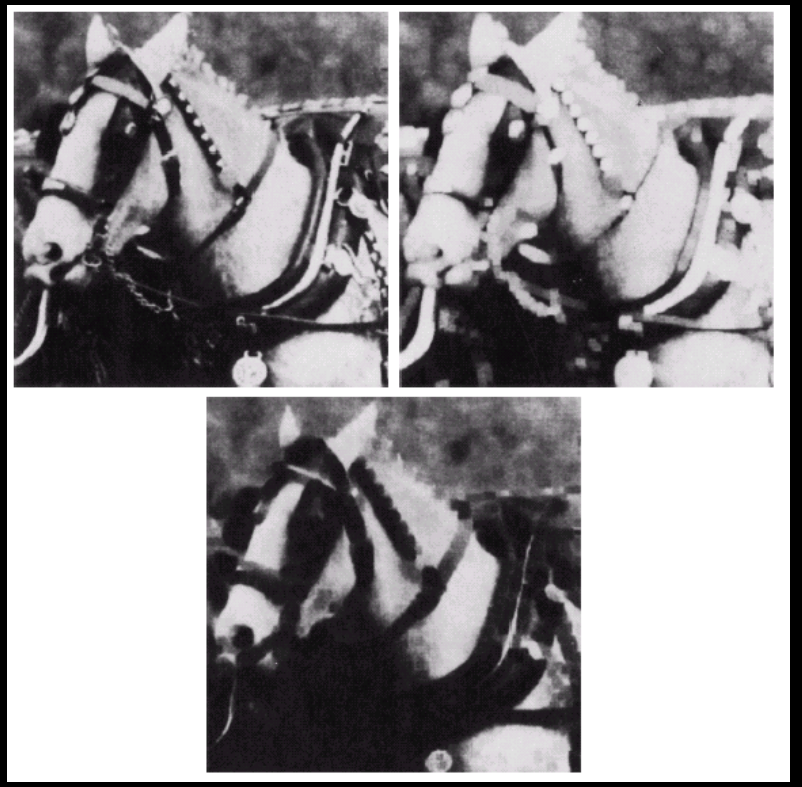
DE zhrnutie

D:

- Jasnejší obrázok
- Redukuje (odstraňuje) tmavé detaily

E:

- Tmavší obrázok
- Redukuje (odstraňuje) svetlé detaily



Otvorenie

$$A \circ B = (A \ominus B) \oplus B$$

Otvorenie

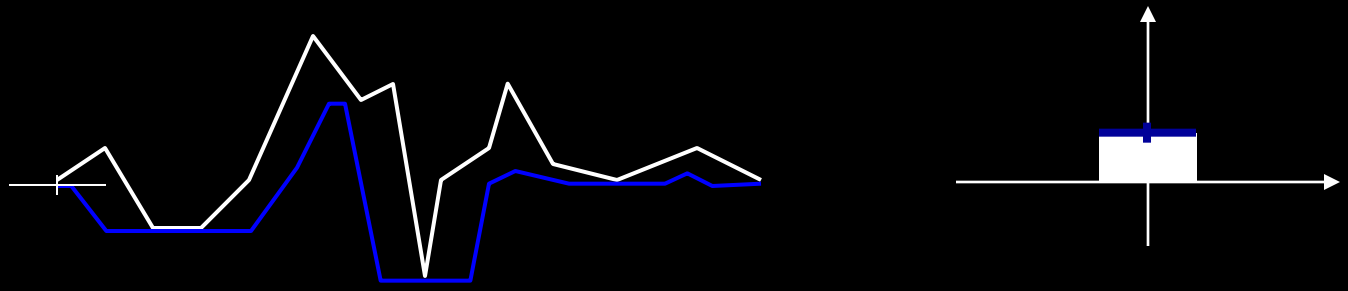
Uzavretie

$$A \bullet B = (A \oplus B) \ominus B$$

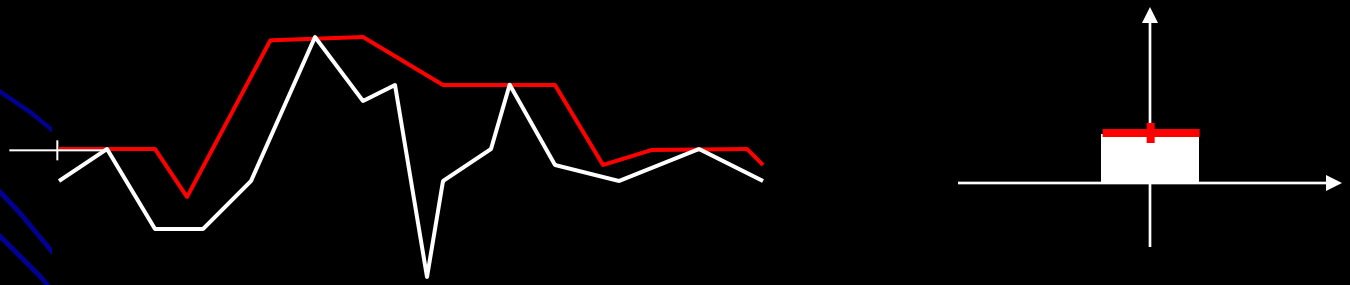
Uzavretie

$$A \circ B = (A \ominus B) \oplus B$$ OU $$A \bullet B = (A \oplus B) \ominus B$$

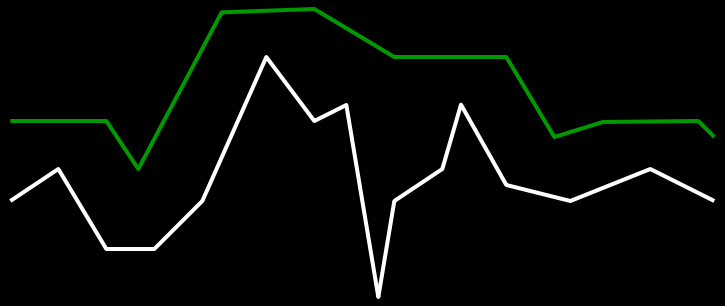
otvorenie



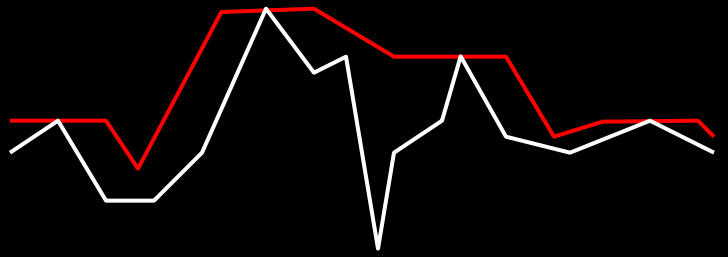
uzavretie



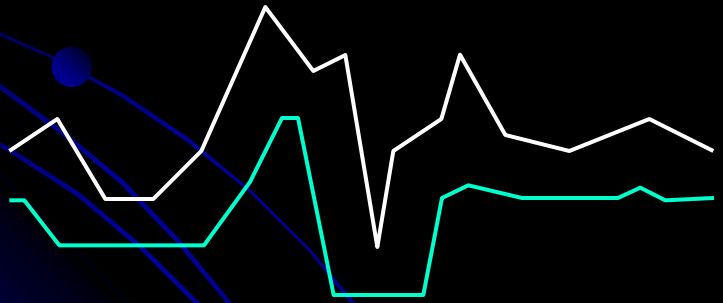
Zhrnutie



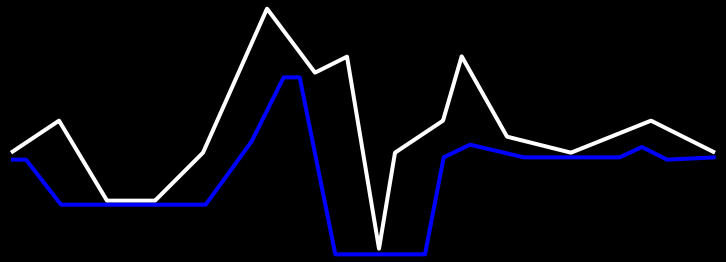
D



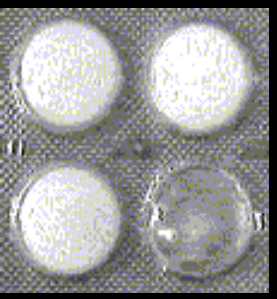
U

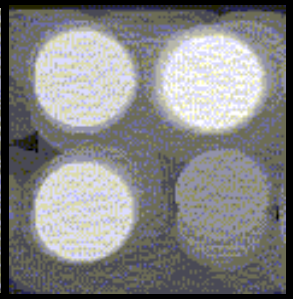


E



O

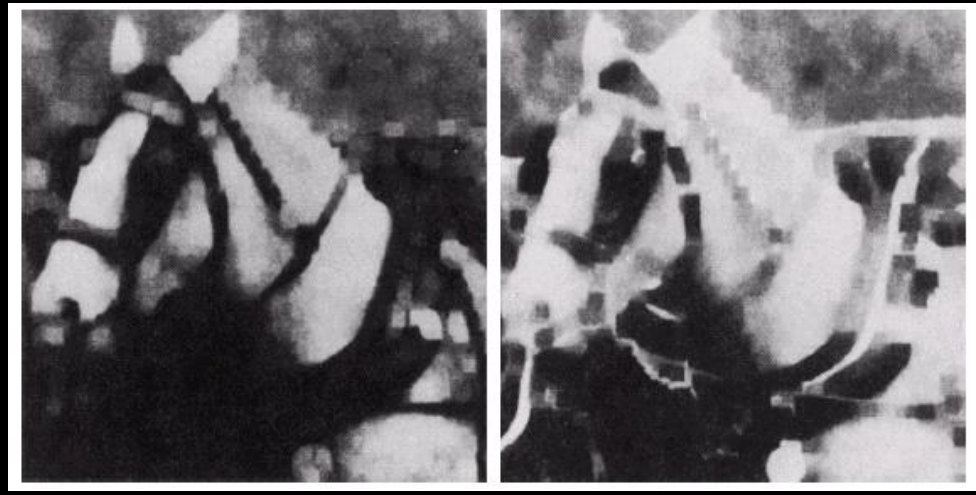




OU zhrnutie

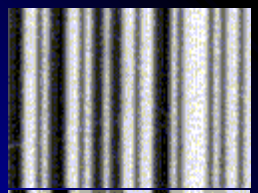
O: odstraňuje malé svetlé objekty

odstraňuje šum

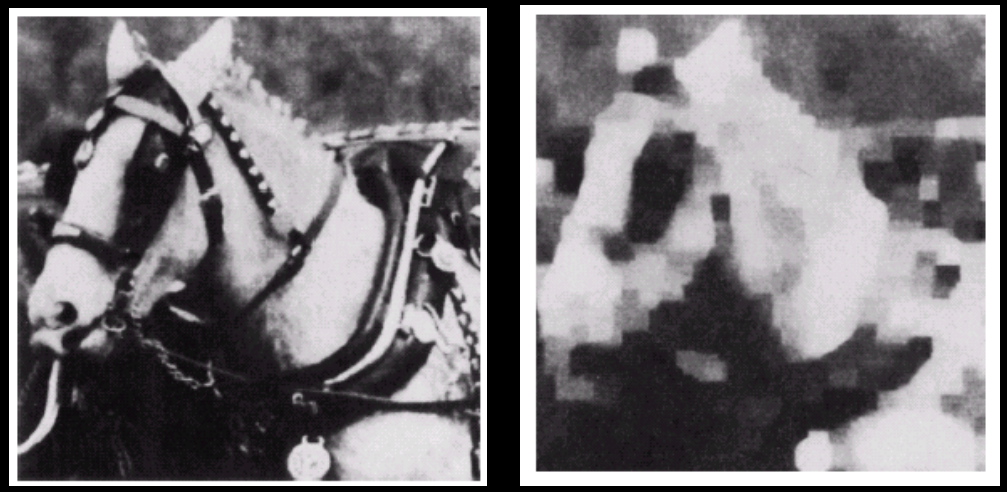


U: spája svetlé objekty

redukuje malé tmavé oblasti



Príklad použitia



$(A \circ B) \bullet B$ – filtrácia obrazu

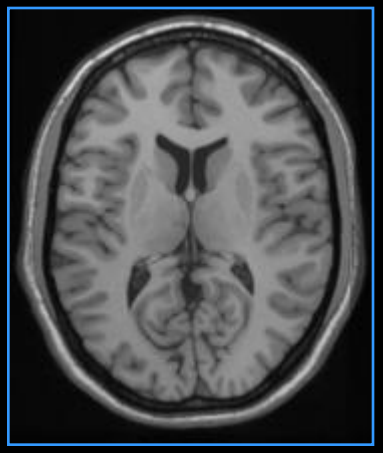
Morfologický gradient

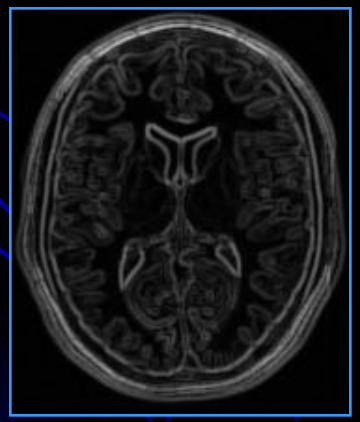
Šedotónové obrazy

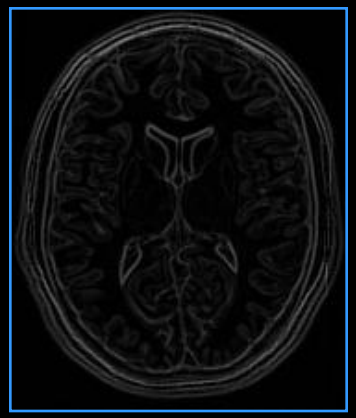
$$grad(F) = \frac{1}{2}(A \oplus B) - (A \ominus B)$$

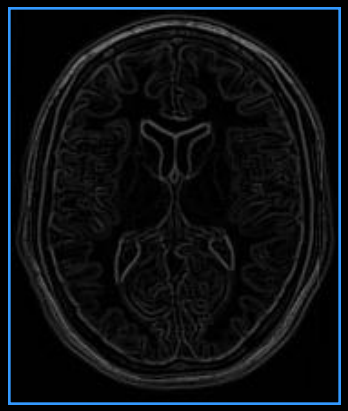
tiež externý a interný gradient

Morfologický gradient



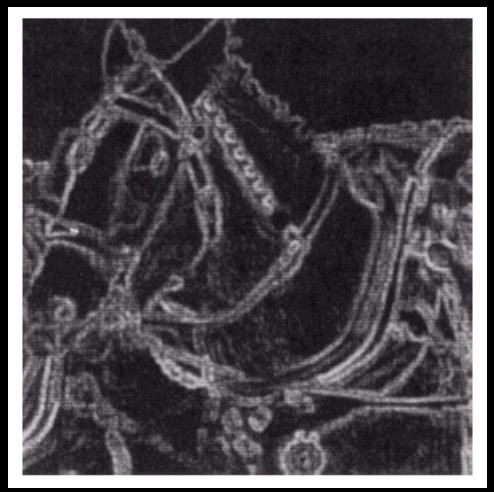






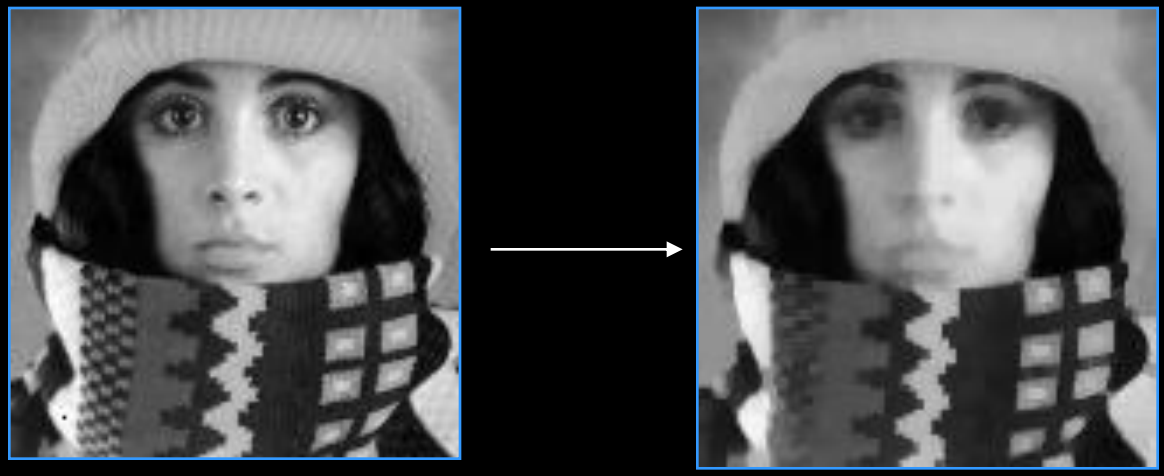
Štandard Externý Interný

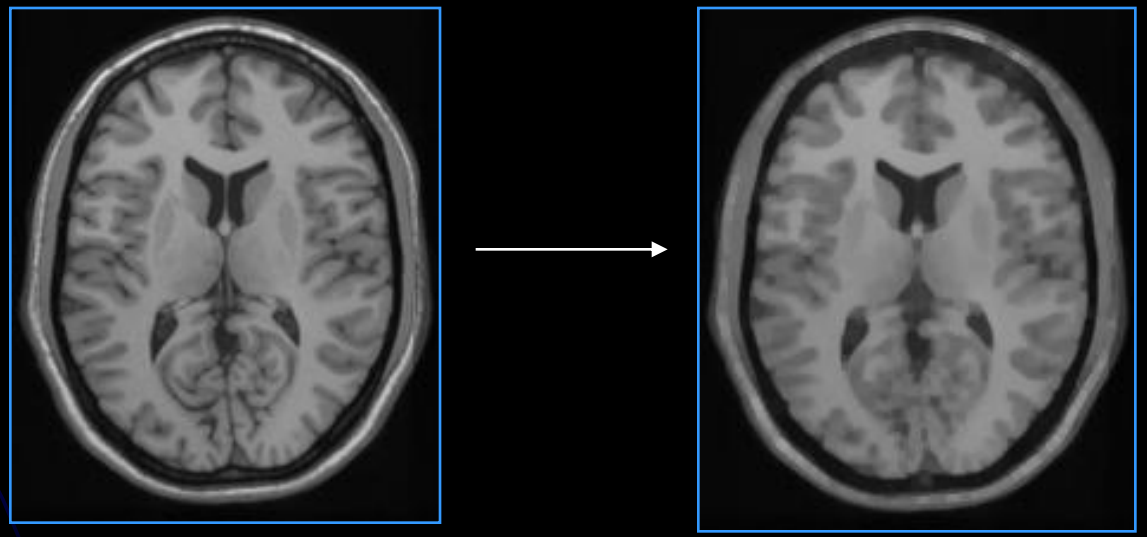
Morfologický gradient



Morfologické vyhladzovanie

$(A \circ B) \bullet B$





Top-hat transformácia

Nástroj na segmentáciu –
výber svetlých (tmavých) objektov
z nekonštantného pozadia

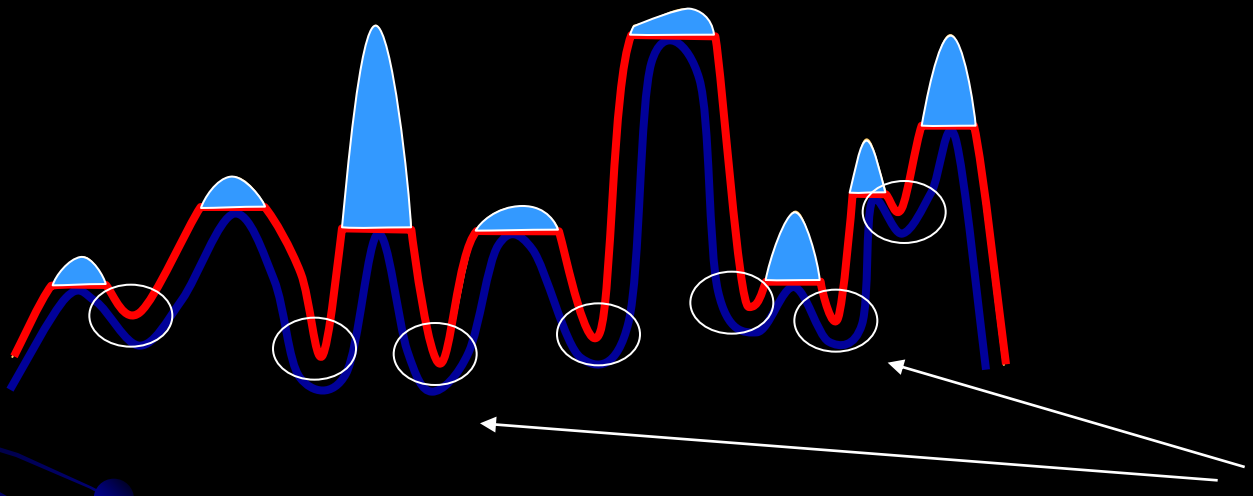
White Top-hat Transform (WTT):

$$A - (A \circ B)$$

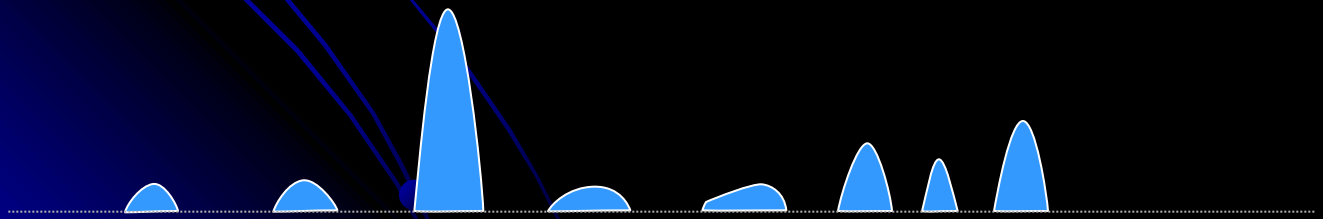
Black Top-hat Transform (BTT):

$$(A \bullet B) - A$$

Top-hat

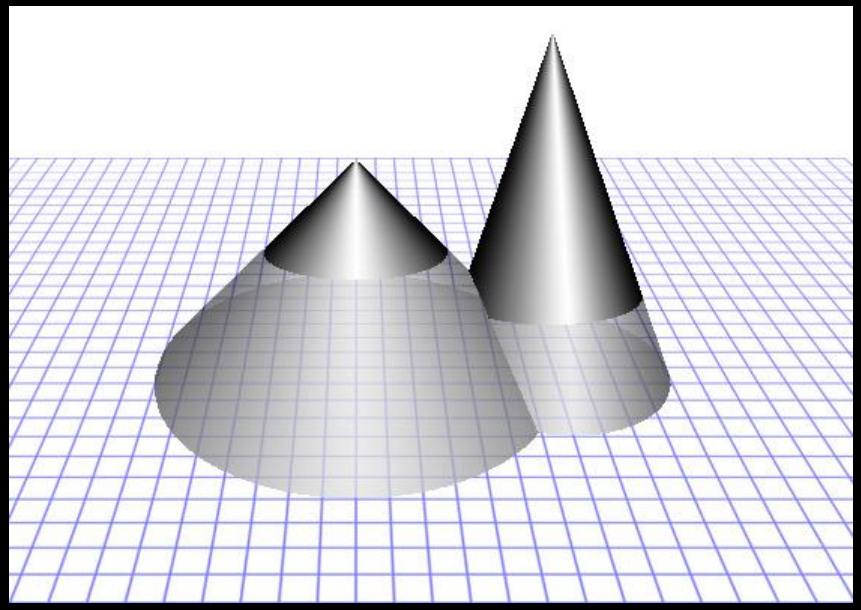


BTT



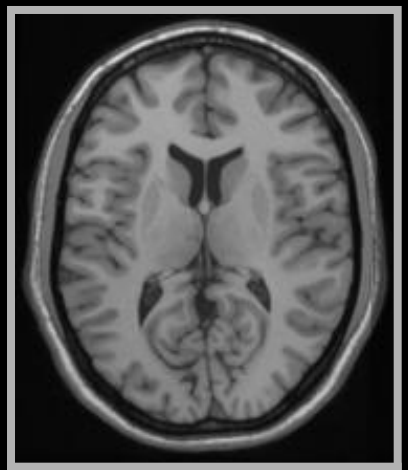
WTT

Top-hat



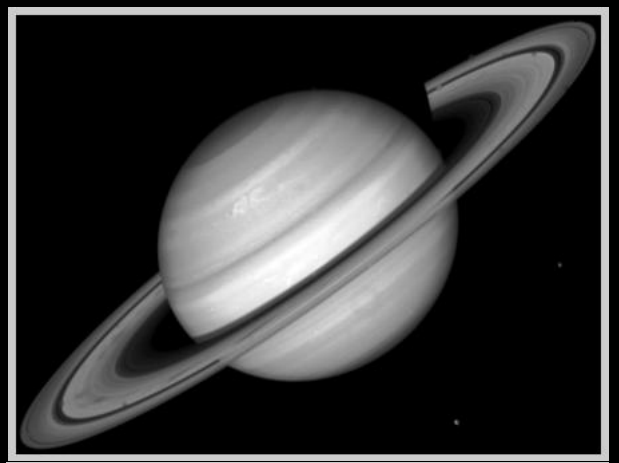
tophat + otvorenie = originál

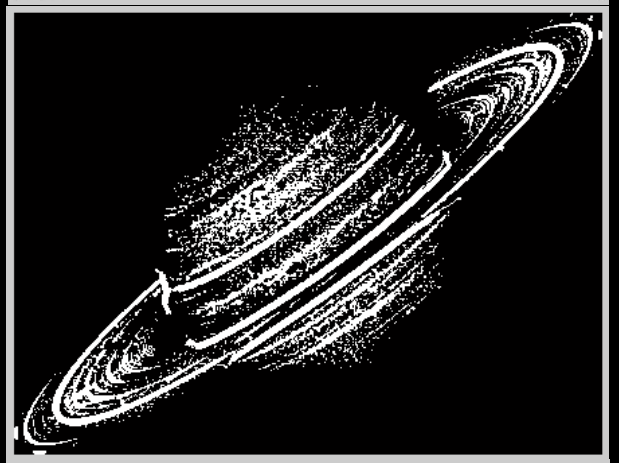
Top-hat

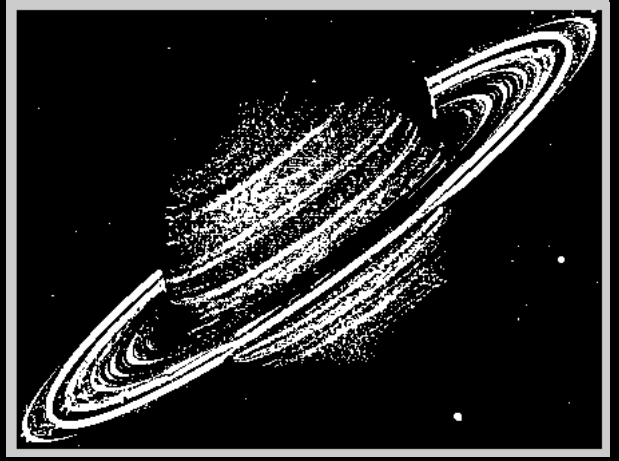


BTT

WTT

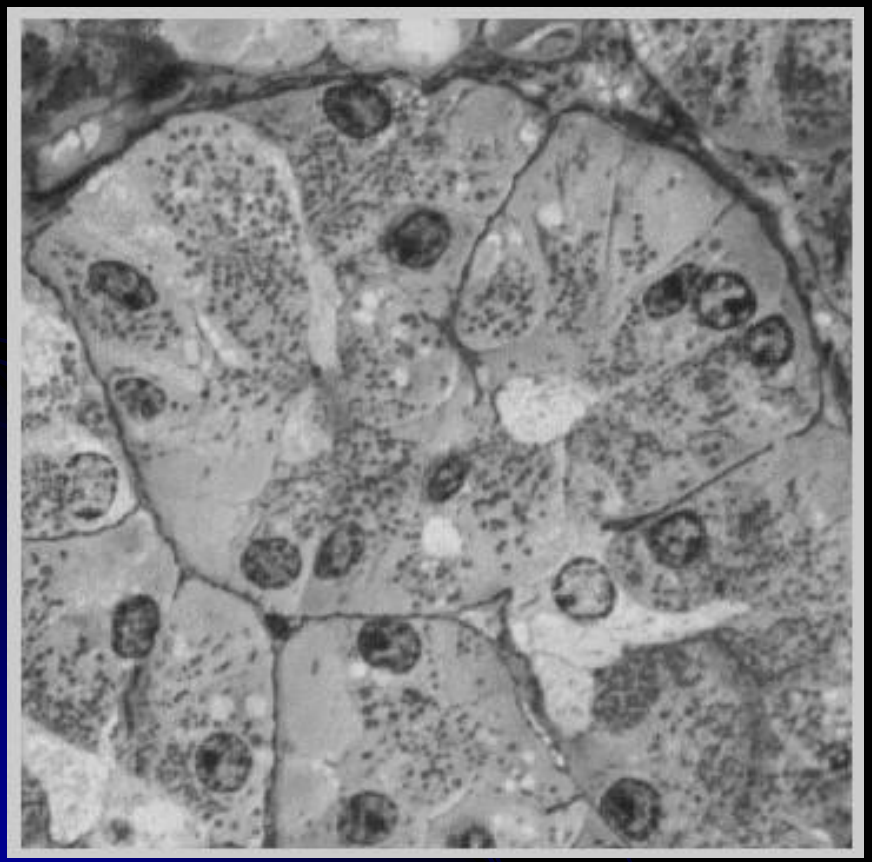


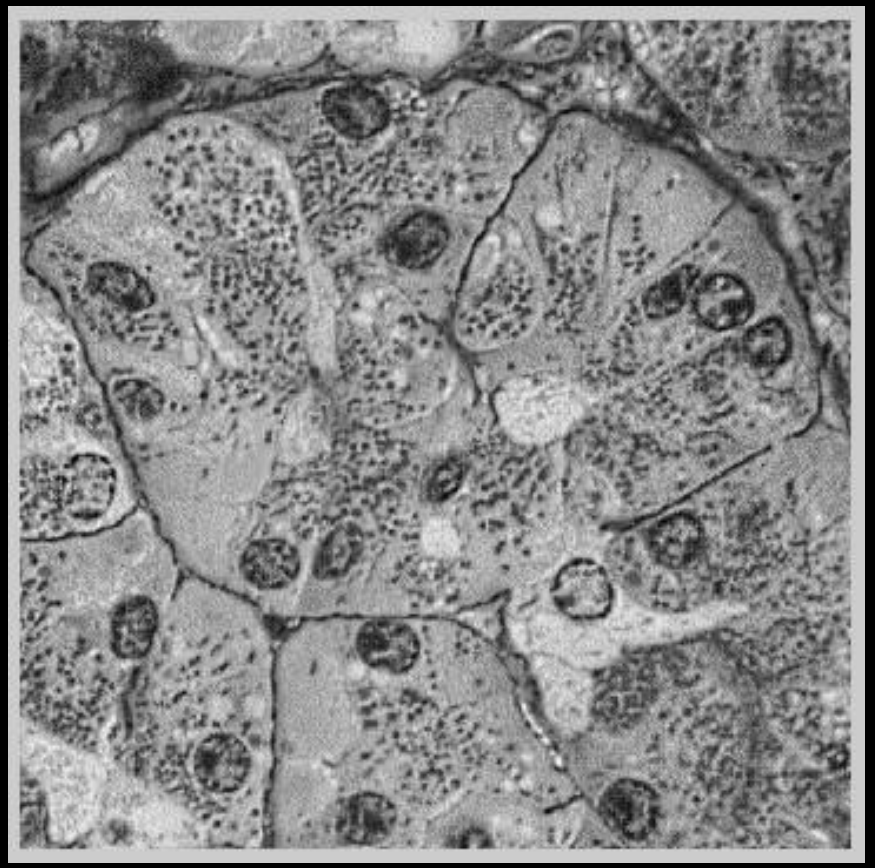




Zlepšenie kontrastu

(A + WTT) – BTT





White top hat

StructuringElement size 3 x 3

White top hat

StructuringElement size 3 x 3

Black top hat

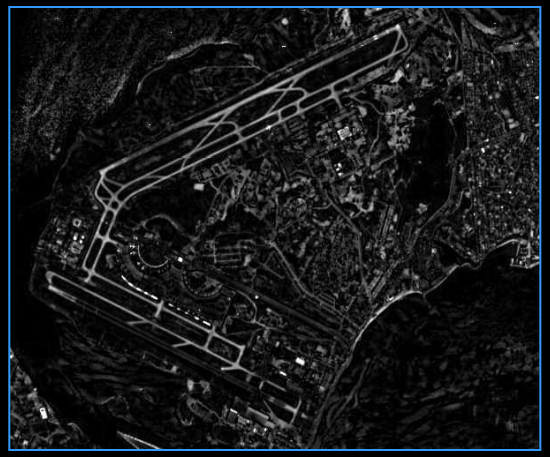
COLOR2GRAY

StructuringElement size 3 x 3

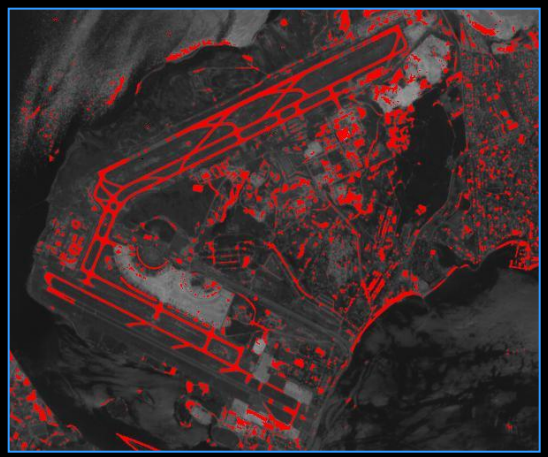
Black top hat

Aplikácia: hľadanie ranvejí

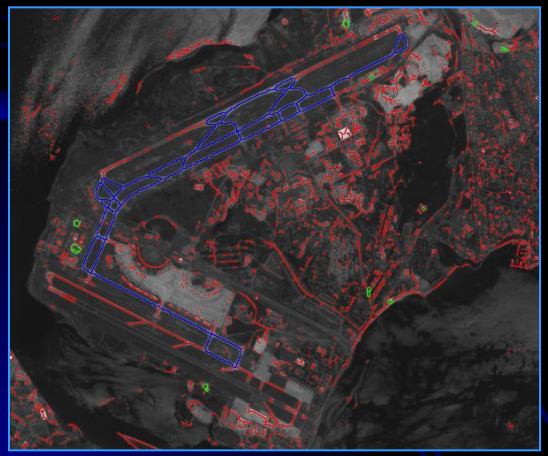
Originál



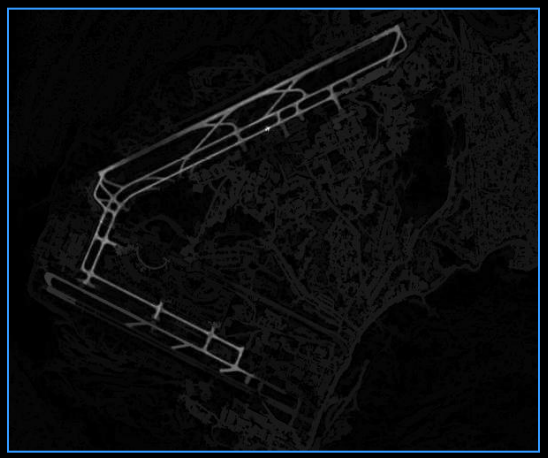
WTT



Prahovanie



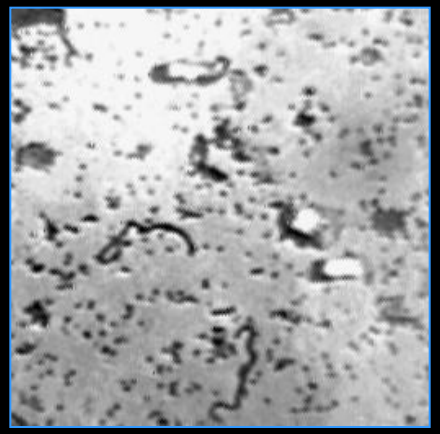
Nájdeme dlhé objekty



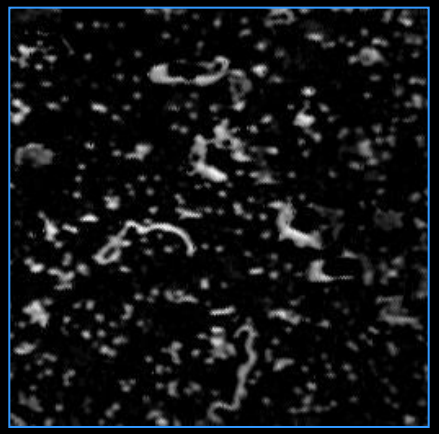
Rekonštrukcia

Výsledok

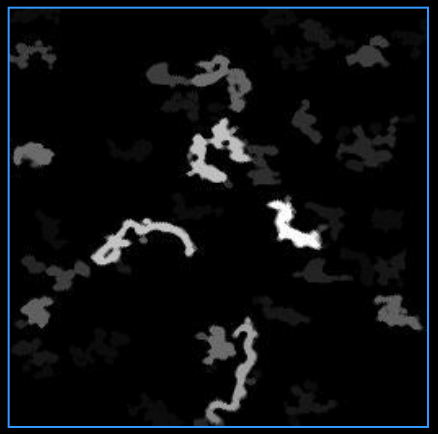
Aplikácia 2: hľadanie filariálnych červov



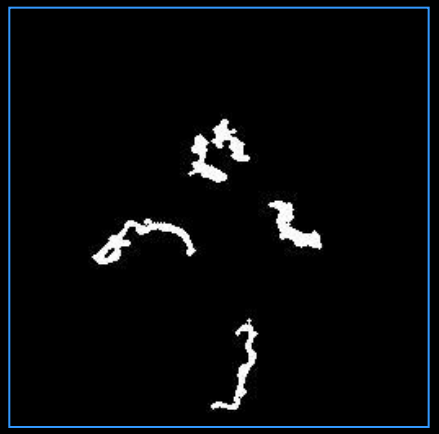
Originál



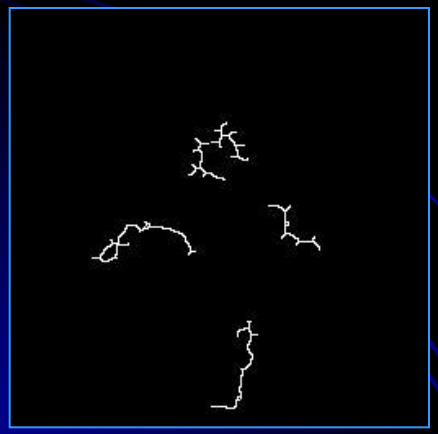
BTT



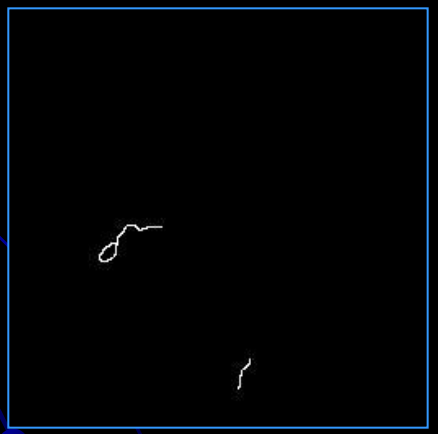
Odstránenie šumu



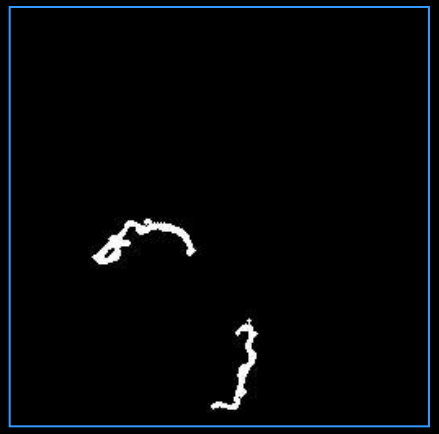
Prahovanie



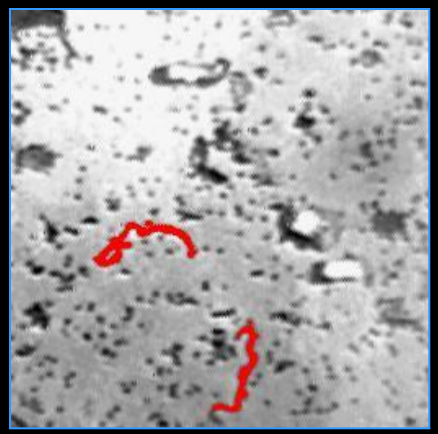
Kostra



Zmazanie krátkych objektov



Rekonštrukcia

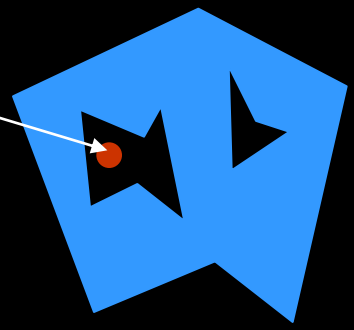


Výsledok

Vypĺňanie oblastí

x=X_0 vnútorný štartovací bod

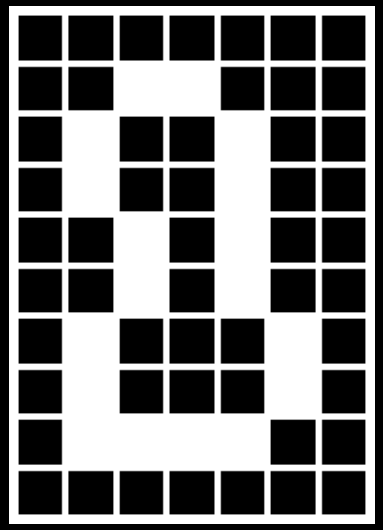
$$X_k = \left(X_{k-1} \oplus B\right) \cap A^C, k = 1,2,3,\ldots$$



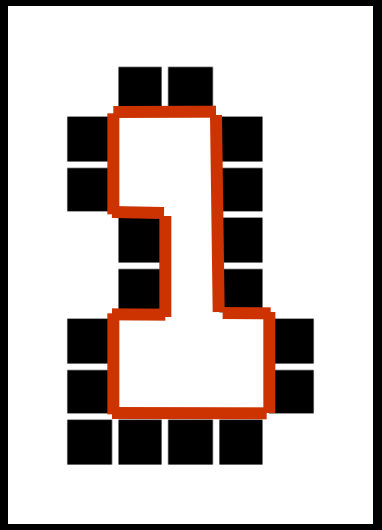
kým X_k=X_{k-1}

Vyplnená oblasť $A \cup X_k$

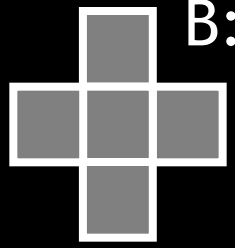
Podmienená dilatácia



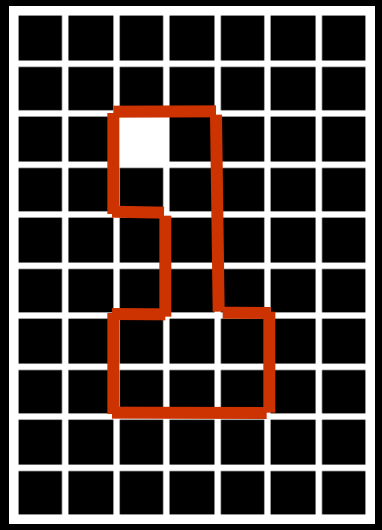
A



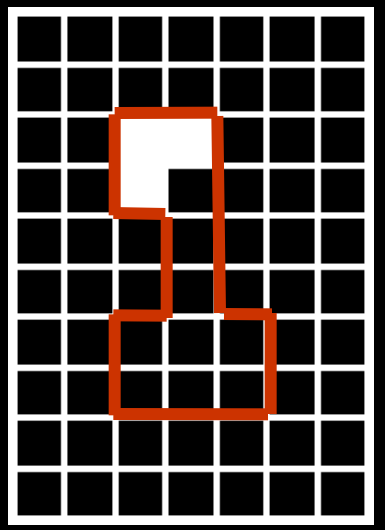
A^c



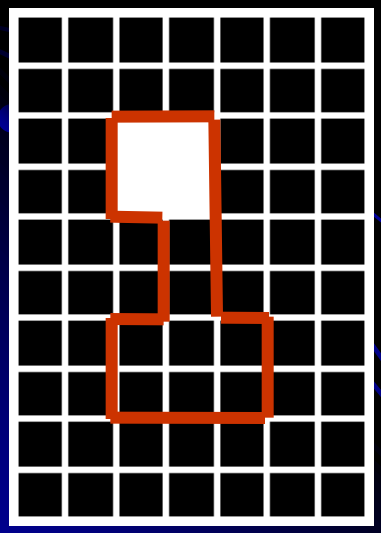
B: ŠP – závisí od susednosti



X_0

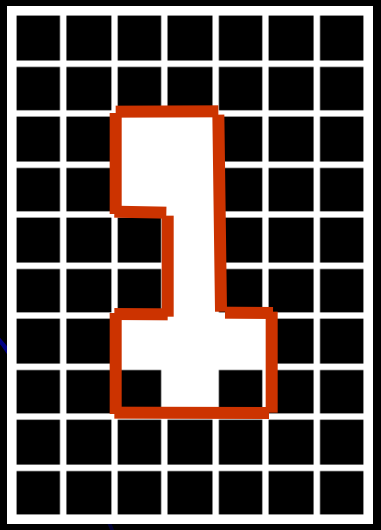


X_1

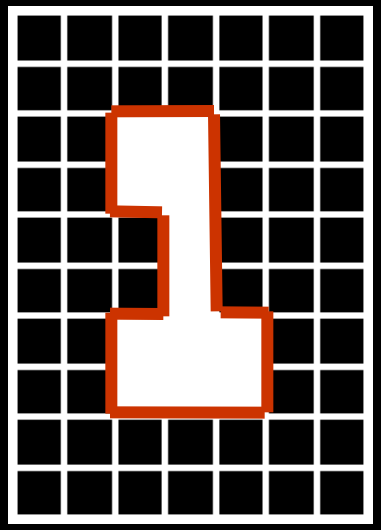


X_2

…

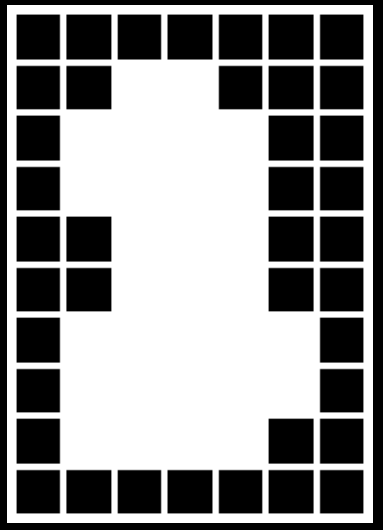


X_6



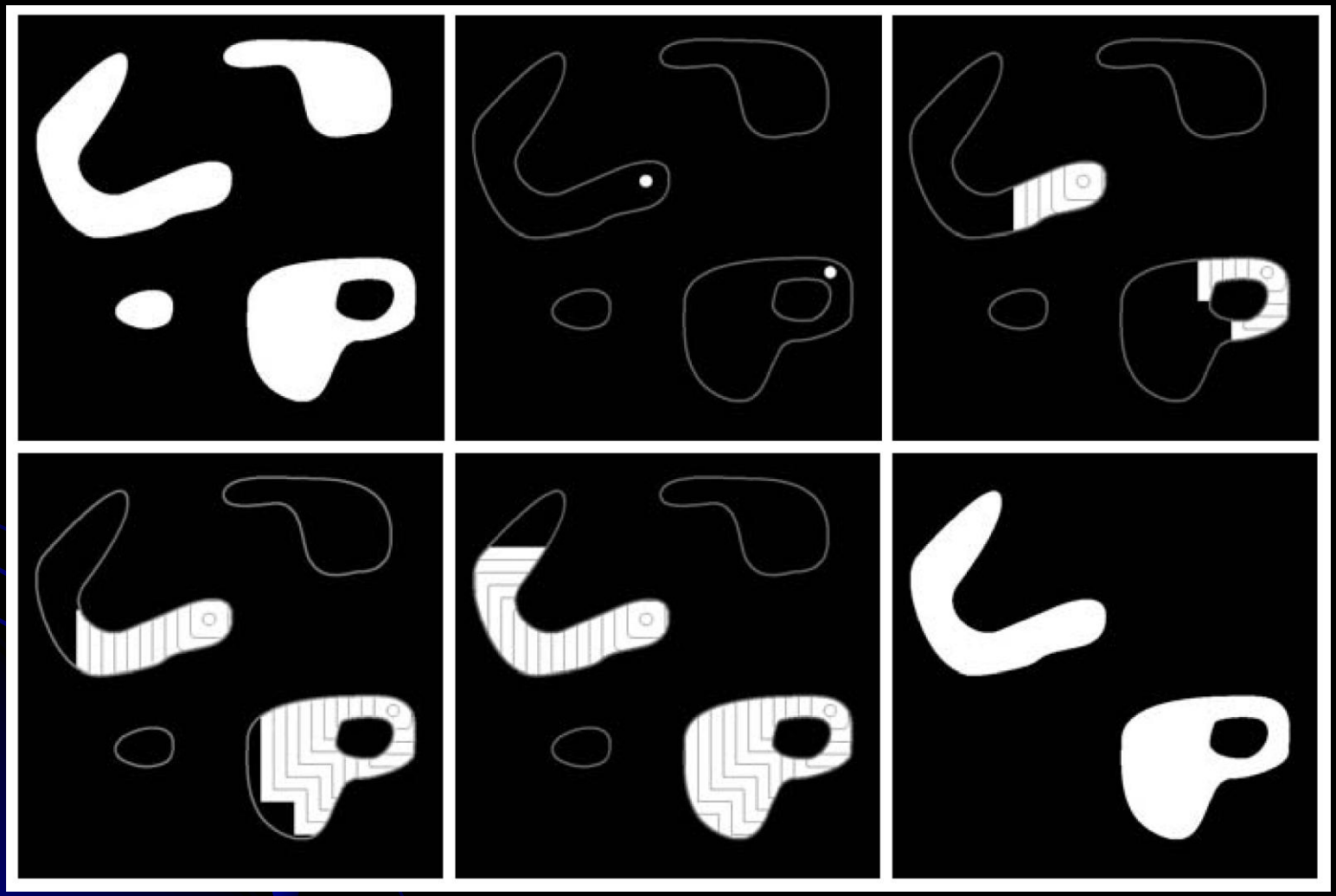
X_7

→



$X_7 \cup A$

Rekonštrukcia



Výber spojitých častí

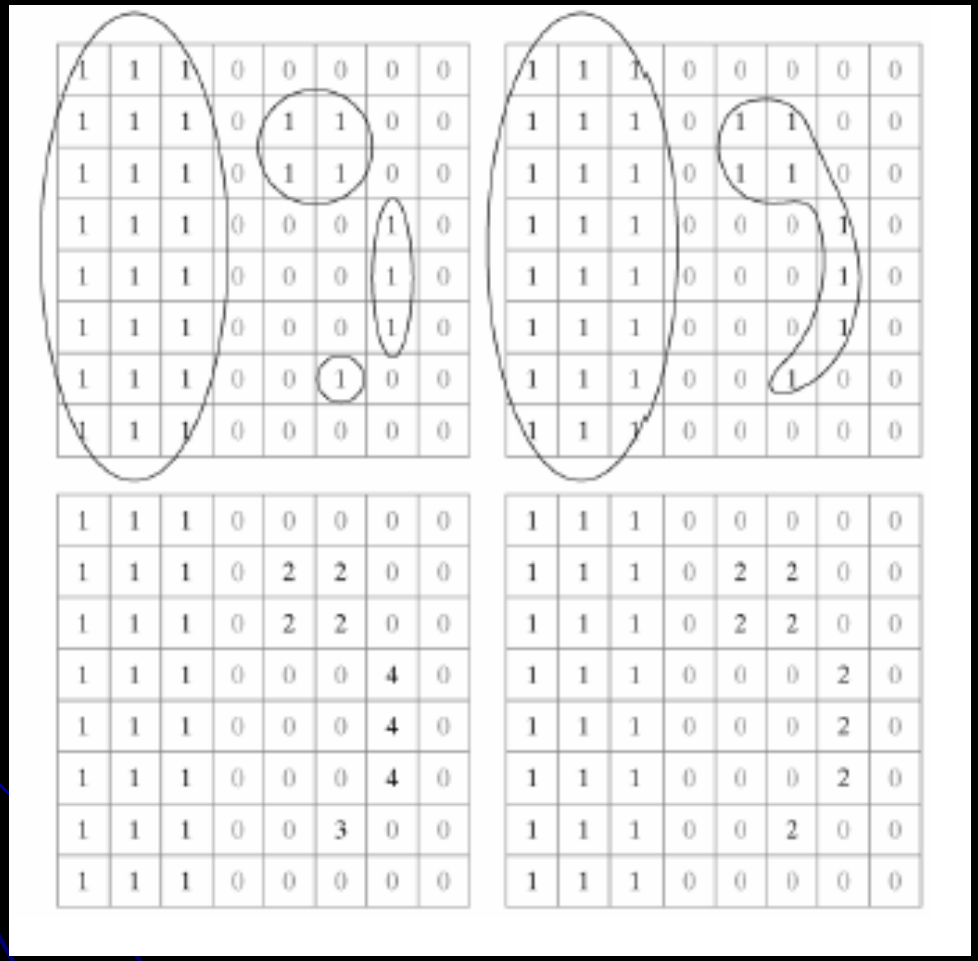
Y – spojitý komponent v množine A
Počiatočný bod $X_0 \in Y$

$$X_k = (X_{k-1} \oplus B) \cap A, k = 1,2,3,...$$

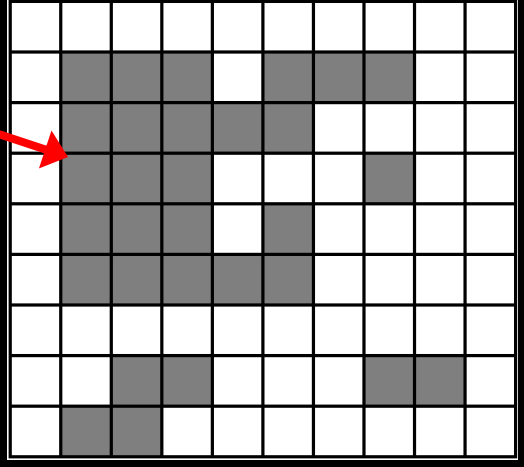
$X_k = X_{k-1} \rightarrow Y = X_k$

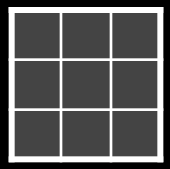
B: ŠP – závisí od susednosti

Výber spojených častí

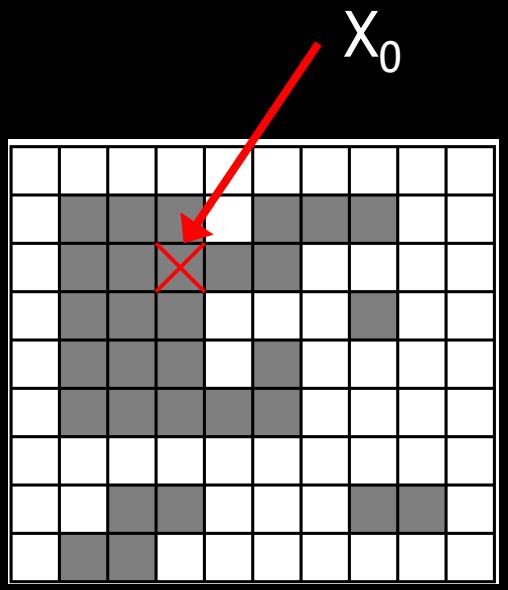


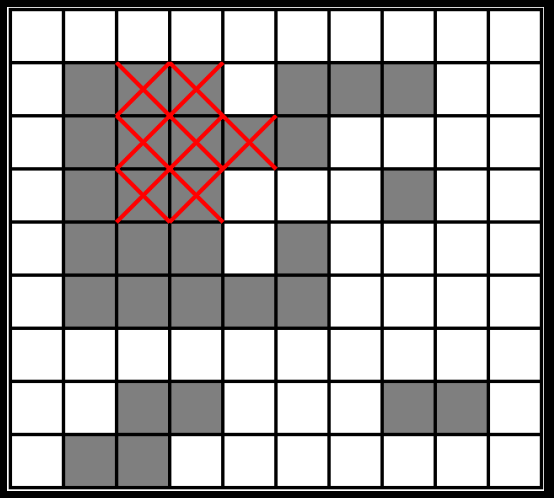
B: ŠP – závisí od susednosti

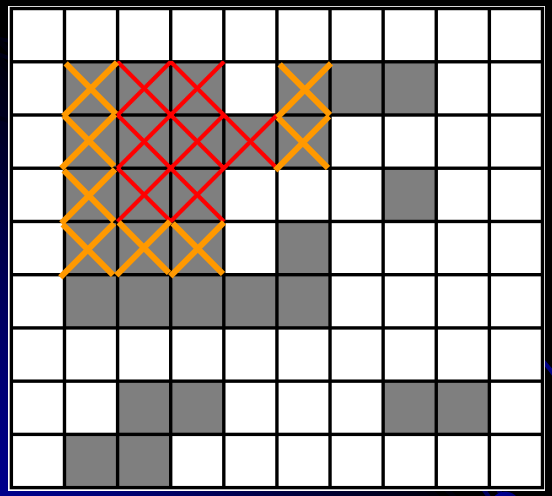


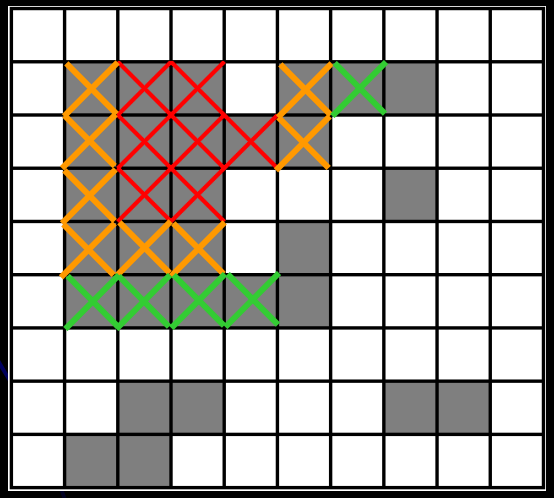


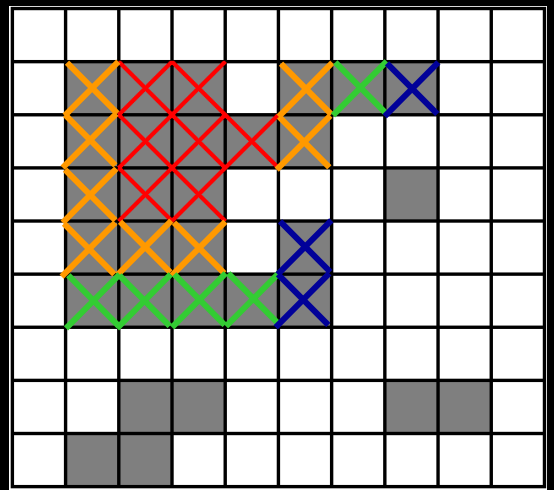
A











Zužovanie

Použitie HMT

$$A \oslash B = A - (A \otimes B)$$

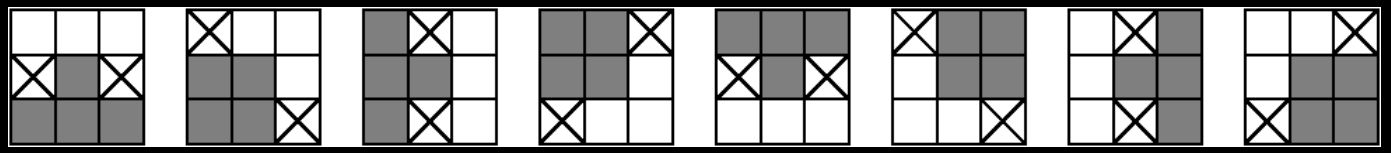
Sekvenčné zužovanie

$((A \oslash B^1) \oslash B^2) \ldots \oslash B^n$

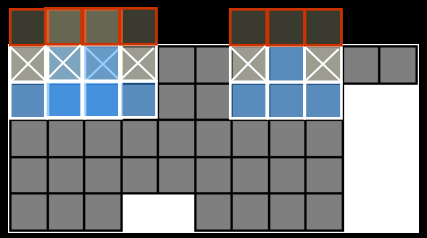
$\{B^1, B^2, \ldots, B^n\}$

Golayova abeceda = typ L, E, M, D, C

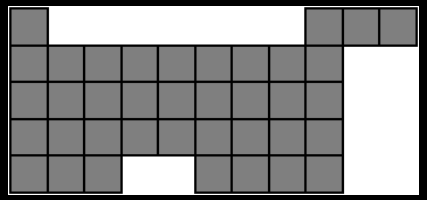
L



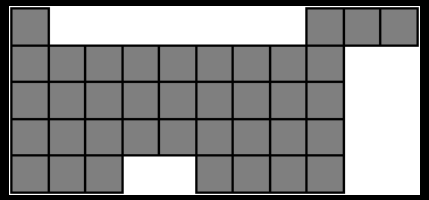
B^1 B^2 B^3 B^4 B^5 B^6 B^7 B^8



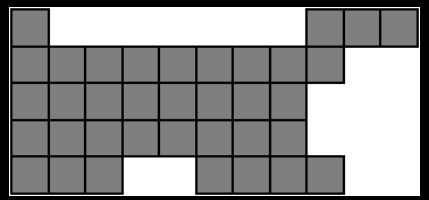
A



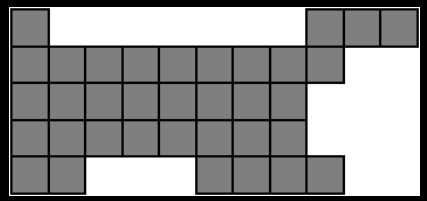
$A \oslash B^1$



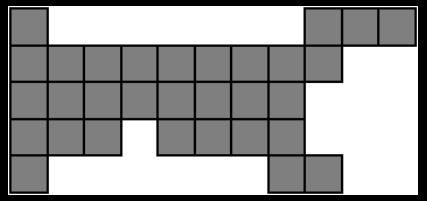
$\ldots \oslash B^2$



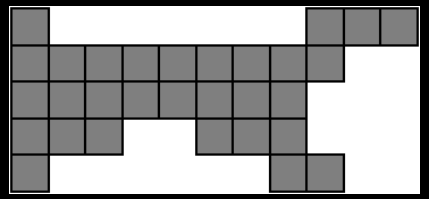
$\ldots \oslash B^3$



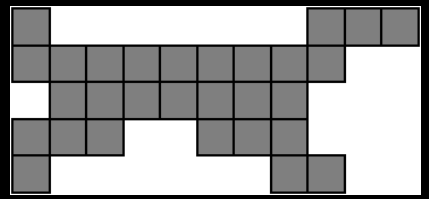
$\ldots \oslash B^4$



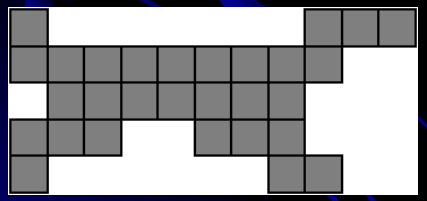
$\ldots \oslash B^5$



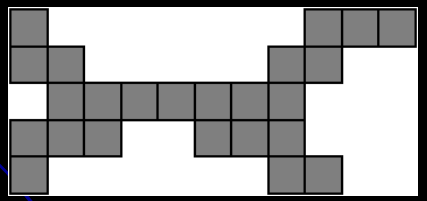
$\ldots \oslash B^6$



$\ldots \oslash B^7$

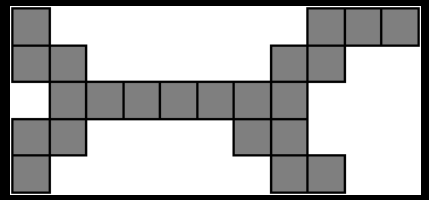


$\ldots \oslash B^8$

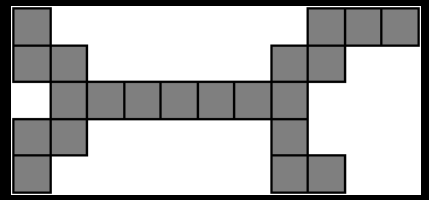


$\ldots \oslash B^1$

...



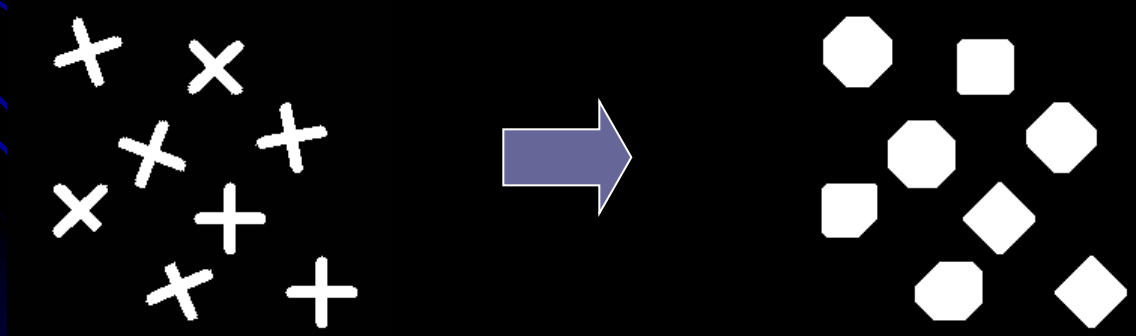
$\ldots \oslash B^5$



$\ldots \oslash B^6$

Rozširovanie

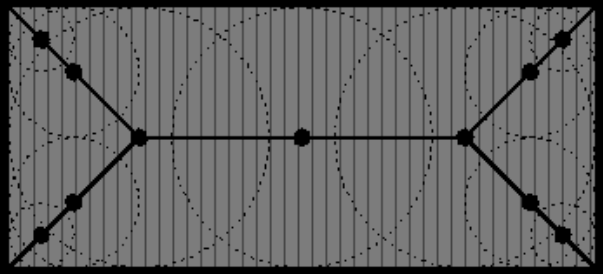
$$A \odot B = A \cup \left(A \otimes B \right)$$



Skeleton – kostra

Kostra – redukcia bodov objektu

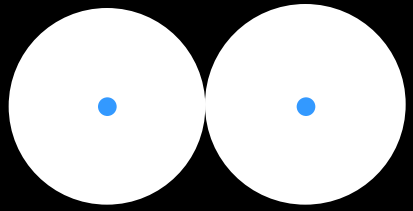
Zachováva topológiu



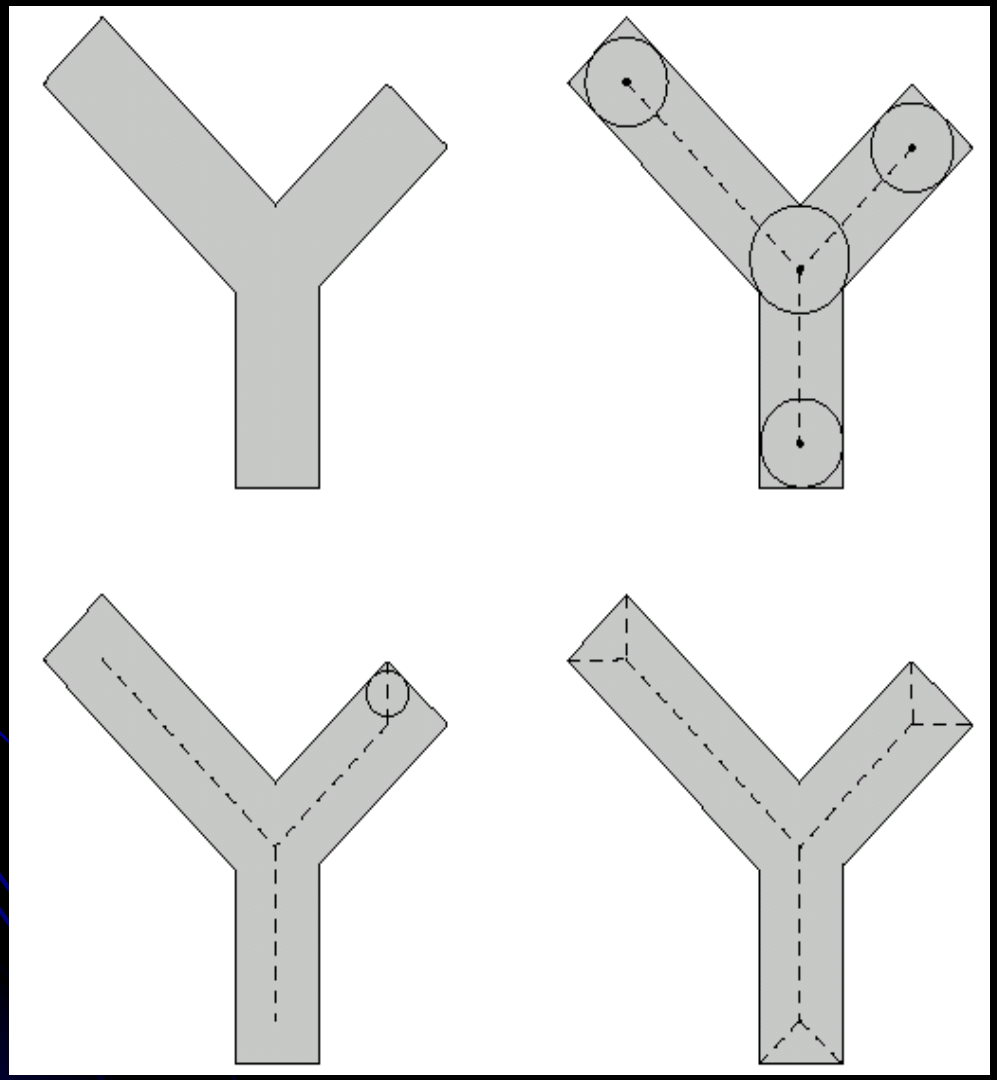
Stredy vpísaných kruhov

$\geq$ 2 dotykové body

Problém?



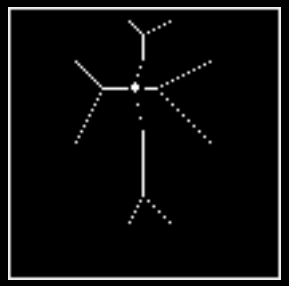
Kostra



Kostra

Podmnožina kostry:

$S_i(F) = (F \ominus^i B) - ((F \ominus^i B) \circ B), \quad i=1,...n$



n – posledná iterácia ($S_{n+1}(F)=\varnothing$)

B kruhový ŠP

Kostra:

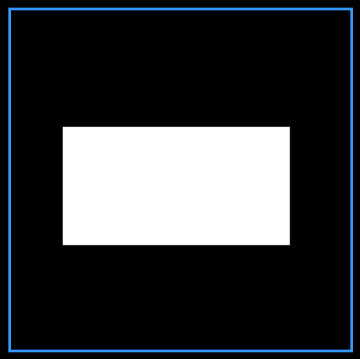
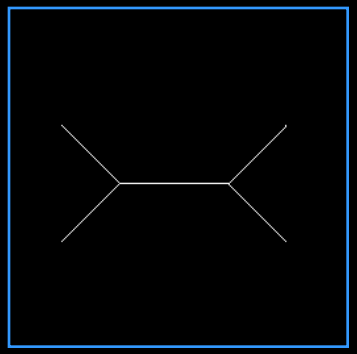
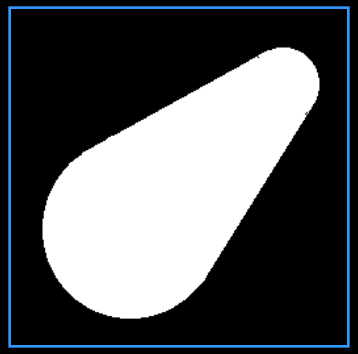
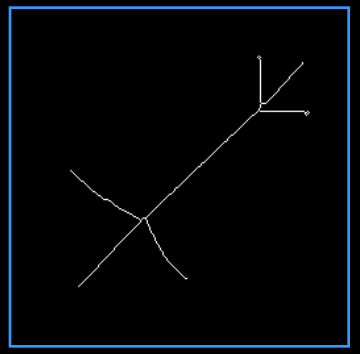
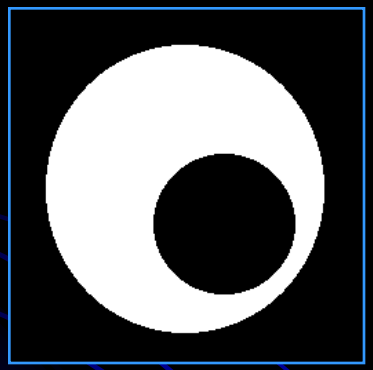
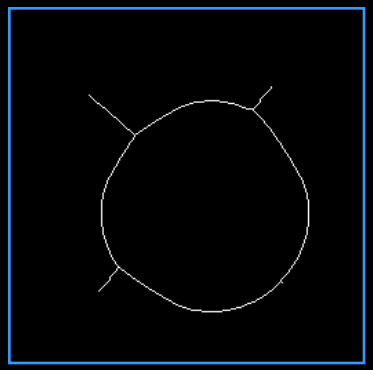
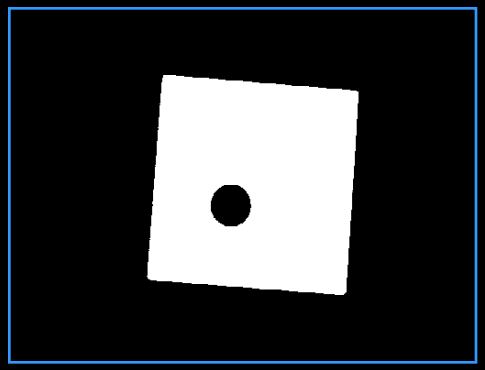
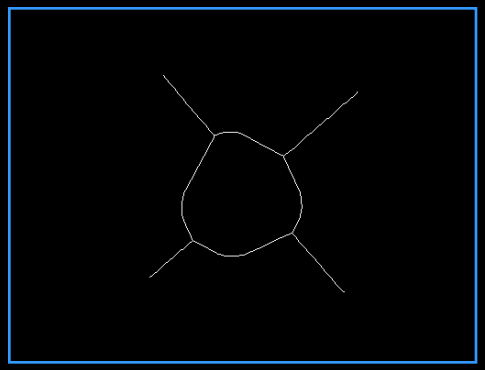
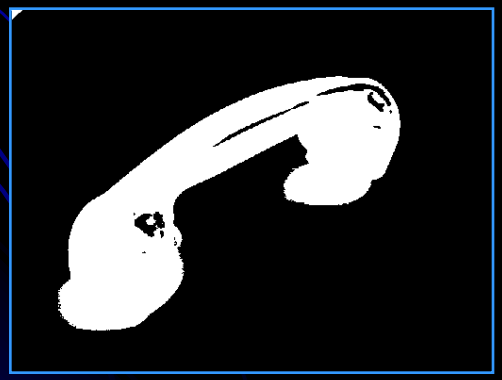
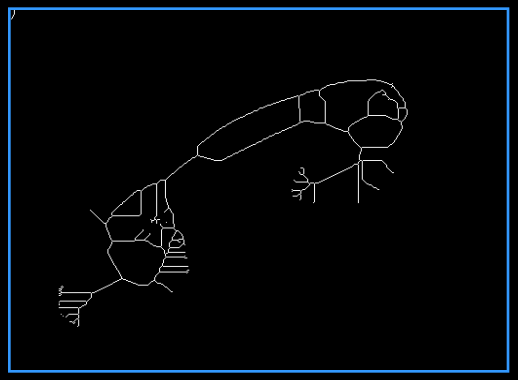
$$S(F) = \bigcup_{i=0}^{n} S_i(F)$$

Kostra

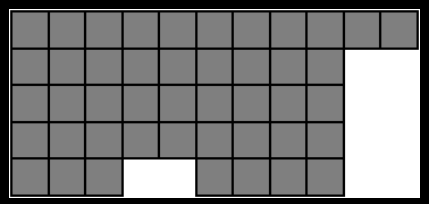
Rekonštrukcia: ak poznáme $S_i(F)$, ŠP B a n:

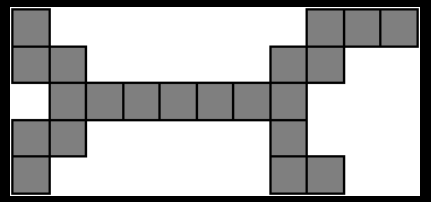
$$F = \bigvee_{i=0}^{n} \left(S_i(F) \oplus^i B \right)$$

Kostra

Kostra - iné algoritmy

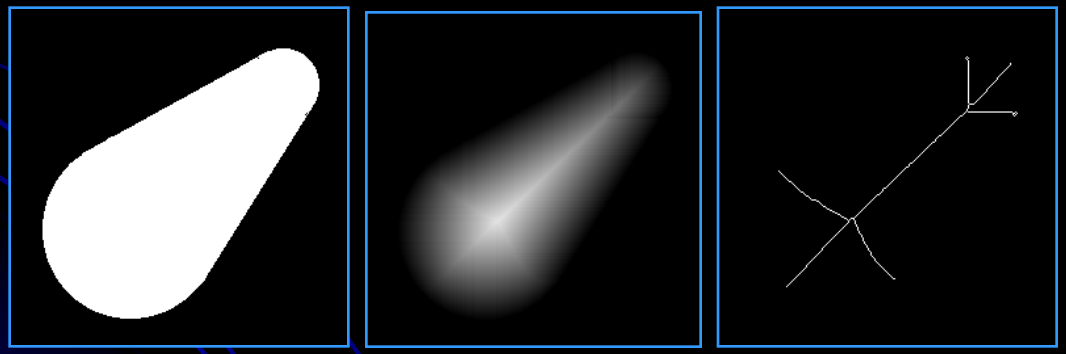




Zužovanie

- Postupné zužovanie
- Zachovávame koncové body a spojnice

Distance Transform



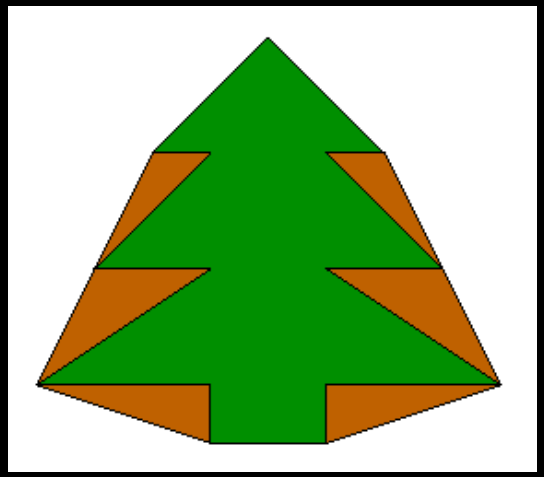
Konvexný obal

Konvexná množina

$x_1, x_2 \in R \Rightarrow \overleftrightarrow{x_1 x_2} \subseteq R$

Konvexný obal (Convex Hull) množiny R

Najmenšia konvexná nadmnožina R



Pruning

Orezávanie – odstraňuje výčnelky

4 kroky

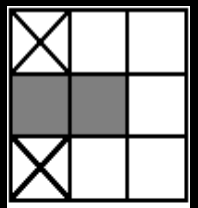
$X_1 = A \oslash \{B^k\}$

$X_2 = \bigcup_{k=1}^{8} (X_1 \otimes B^k)$

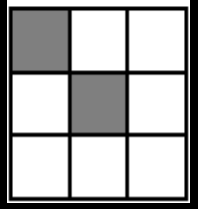
$X_3 = (X_2 \oplus H) \cap A$

$X_4 = X_1 \cup X_3$

B^1

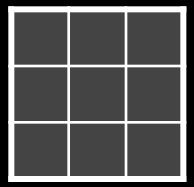


B^5



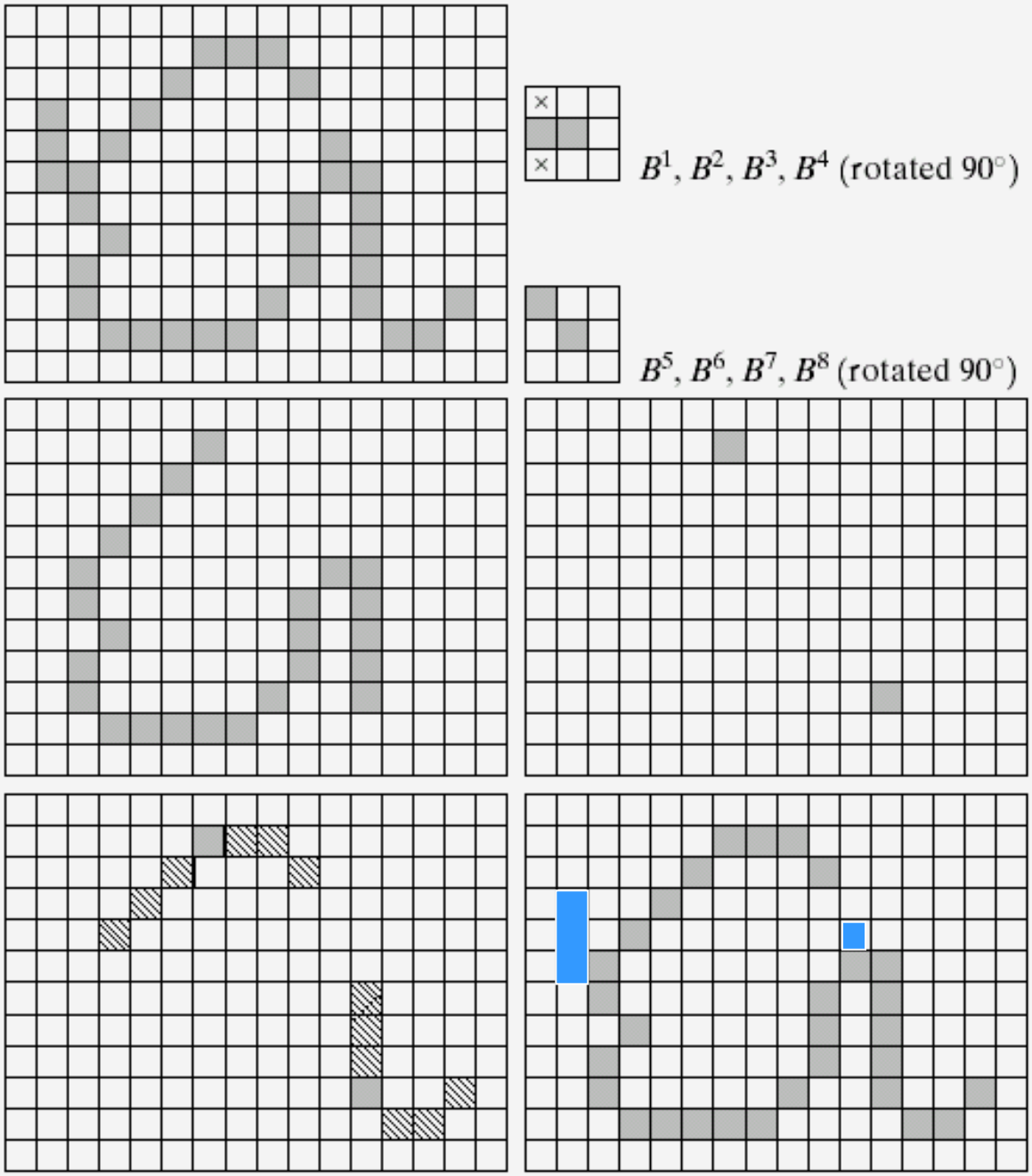
+ otočenie o 90°

H

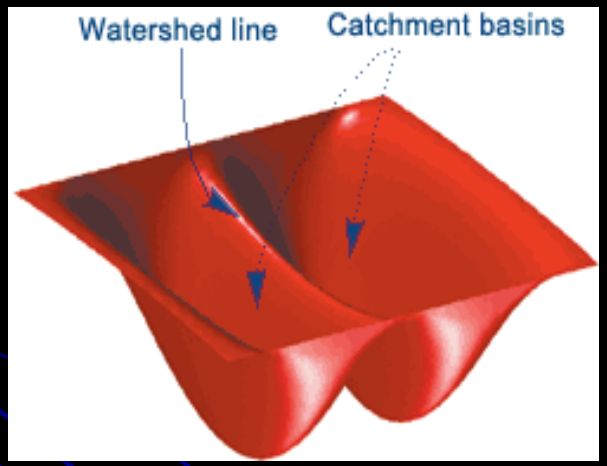


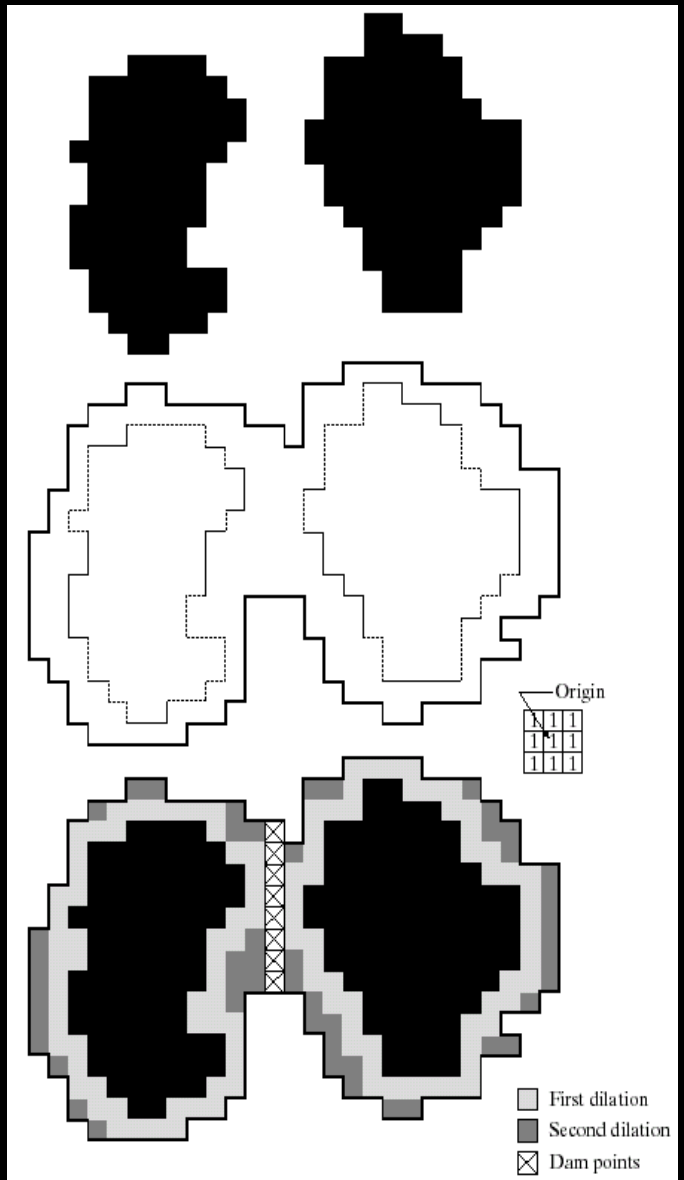
a b
c
d e
f g

FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.



Watershed transformácia





Zhrnutie

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

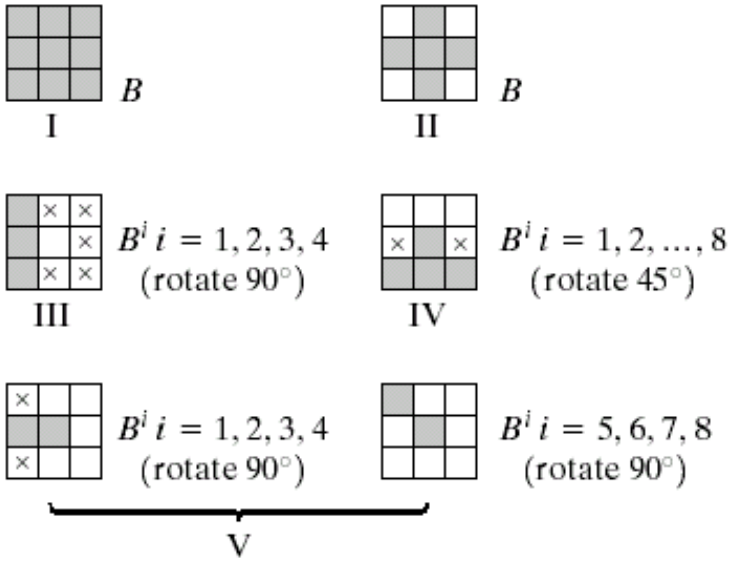


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.