Morfológia

Matematická morfológia

η μορφή = forma, tvar
ο λόγος (τα λόγια) = slovo (slová)

Morfológia = štúdium formy a štruktúry (zvieratá, rastliny)

Matematická morfológia = nástroj na popis komponentov obrazu, tvaru, štruktúry
Základ – teória množín

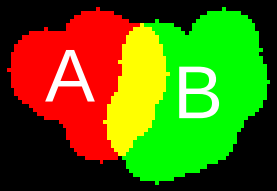
Použitie

predspracovanie
- filtrovanie šumu, zjednodušenie tvarov, ...

segmentácia
- watershed, hrany, obrys, …

popis štruktúry objektov
- kostra, konvexný obal, ...

kvantitatívny popis
- analýza tvaru (area, perimeter, ...),
- granulometria, súvislé oblasti ...

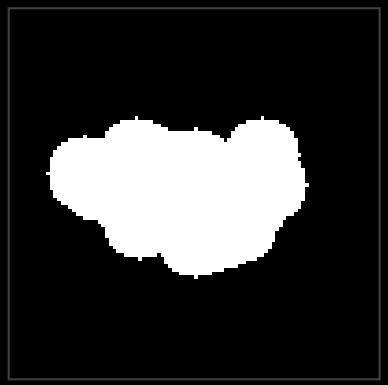


Základné definície

- Prvok množiny $x \in A$
- Prvok nepatriaci množine $x \notin A$
- Prázdna množina – neobsahuje žiadny prvok $\varnothing$
- Disjunktné množiny, ak $A \cap B = \varnothing$
- Podmnožina $A \subseteq B \Leftrightarrow (x \in A \Rightarrow x \in B)$

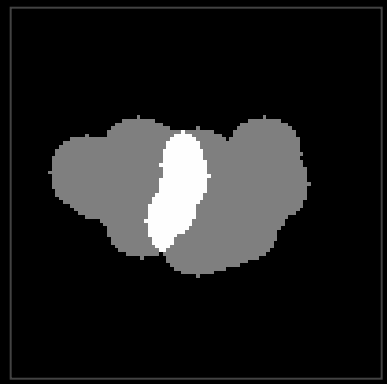
Zjednotenie

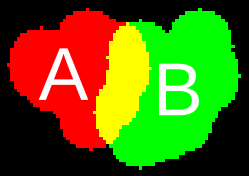
$A \cup B = \{x \mid x \in A \vee x \in B\}$



Prienik

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

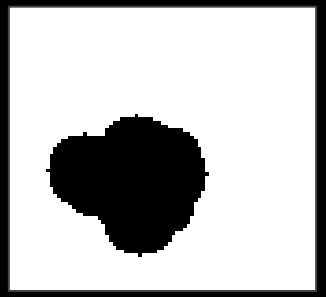




Množinové operácie

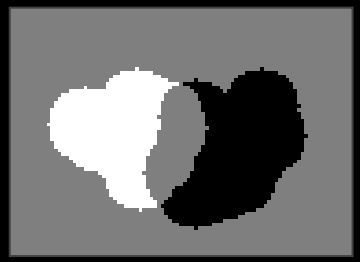
Komplement

$A^c = \{x \mid x \notin A\}$



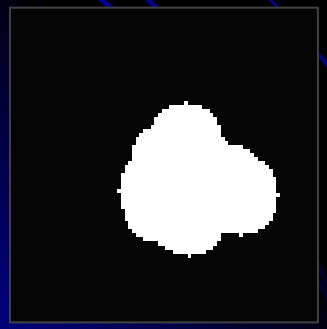
Rozdiel

$A - B = A \cap B^c = \{x \mid x \in A \wedge x \notin B\}$



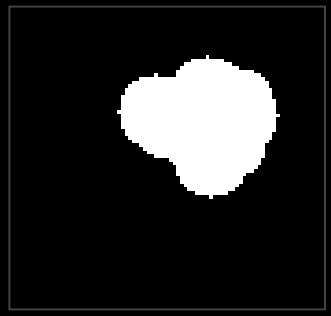
Zrkadlenie

$\hat{A} = \{x \mid x = -a, \forall a \in A\}$



Posunutie

$A_z = A + z = \{x \mid x = a + z, \forall a \in A\}$



Morfologické operácie

Vzťah:

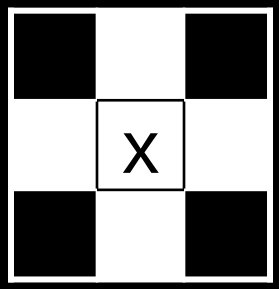
obraz (množina) – štrukturálny prvok

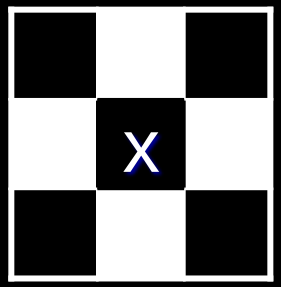
Výsledok:

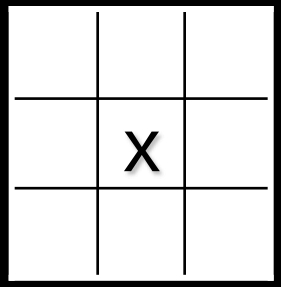
zmena, zmenšenie, zväčšenie množiny

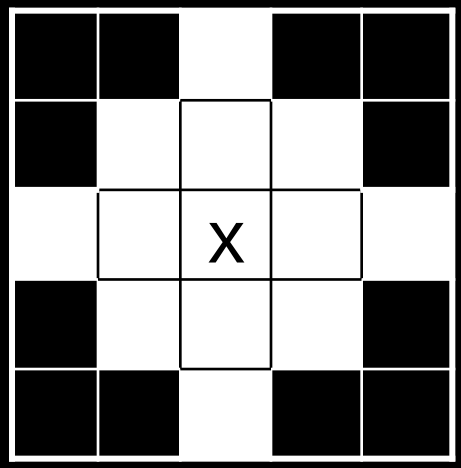
Binárna morfológia

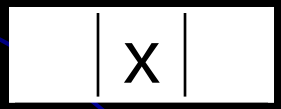
Štrukturálny prvok

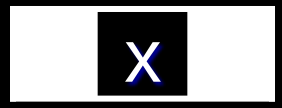












x – počiatok súr. sústavy (0,0)

referenčný bod

Štrukturálny prvok

tvar
veľkosť
orientácia
pozícia vzhľadom k **x**

Závisia od aplikácie, ovplyvňujú výsledok

ŠP je (zvyčajne) oveľa menší ako obraz

Dilatácia


Minkowski

Minkowského súčet $\oplus$

$$A \oplus B = \bigcup_{b \in B} A_b$$

$$A \oplus B = \bigcup_{b \in B} a + b \mid a \in A$$

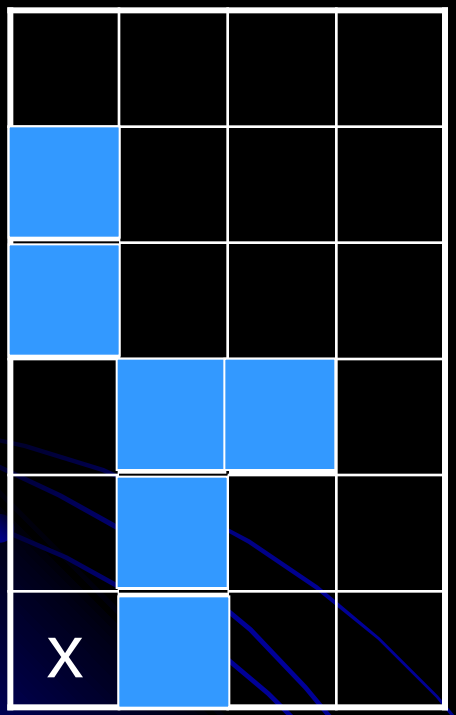
$$= a + b \mid a \in A, b \in B$$

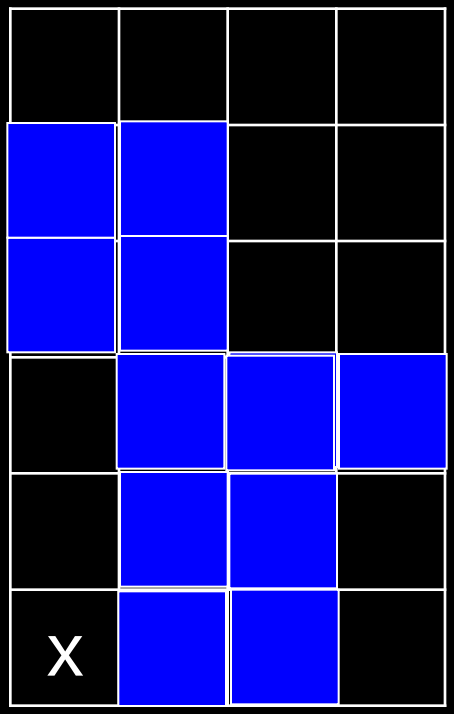
$$A \oplus B = x \mid \hat{B}_x \cap A \neq \varnothing$$

Expanzívna operácia – zväčšuje množinu

$$A \oplus B = \bigcup_{b \in B} A_b$$

Dilatácia



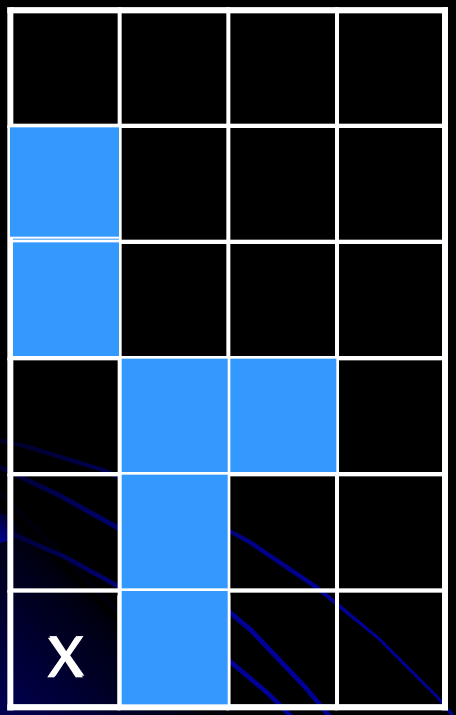


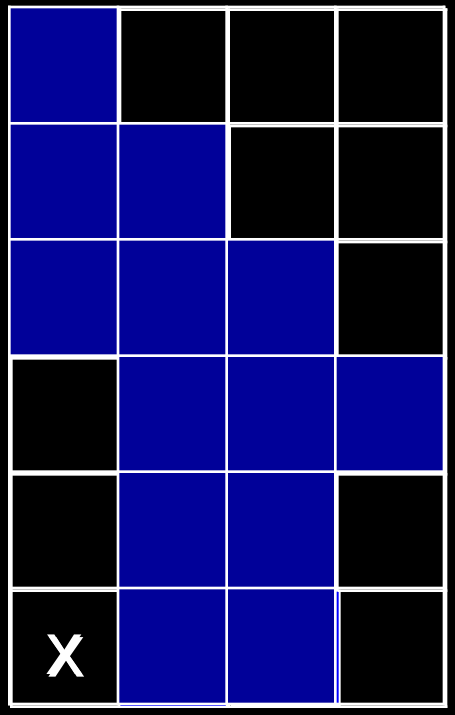
B = x

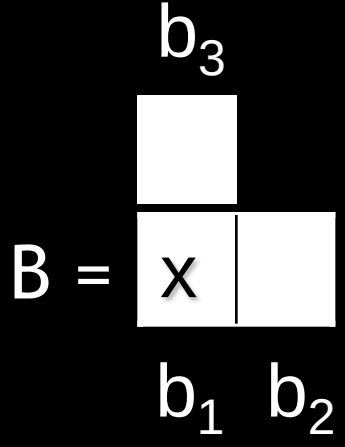
b_1 b_2

$$A \oplus B = \bigcup_{b \in B} A_b$$

Dilatácia

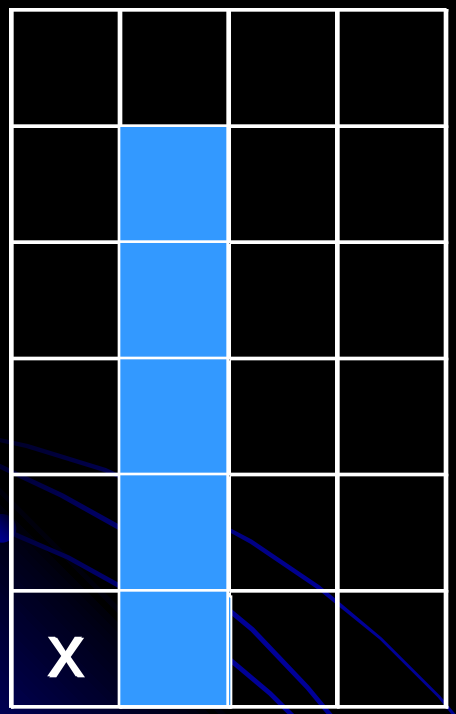


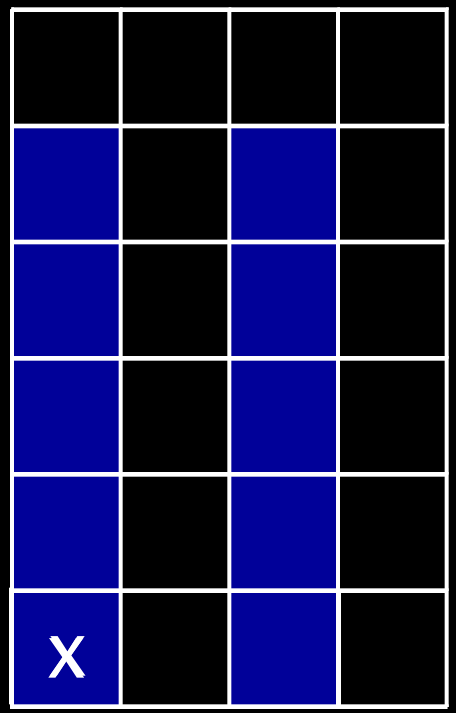


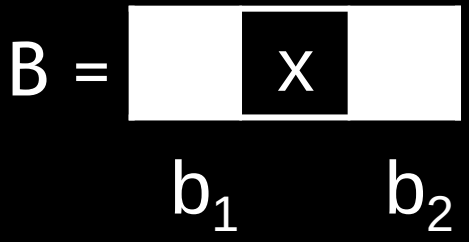


$$A \oplus B = \bigcup_{b \in B} A_b$$

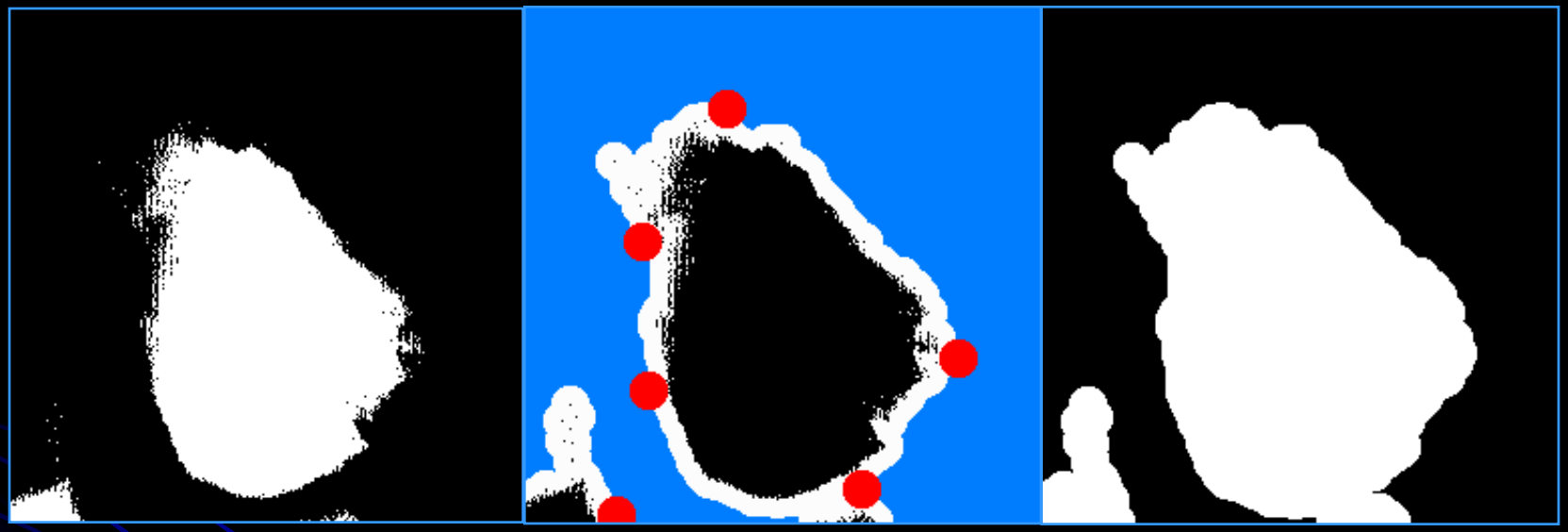
Dilatácia



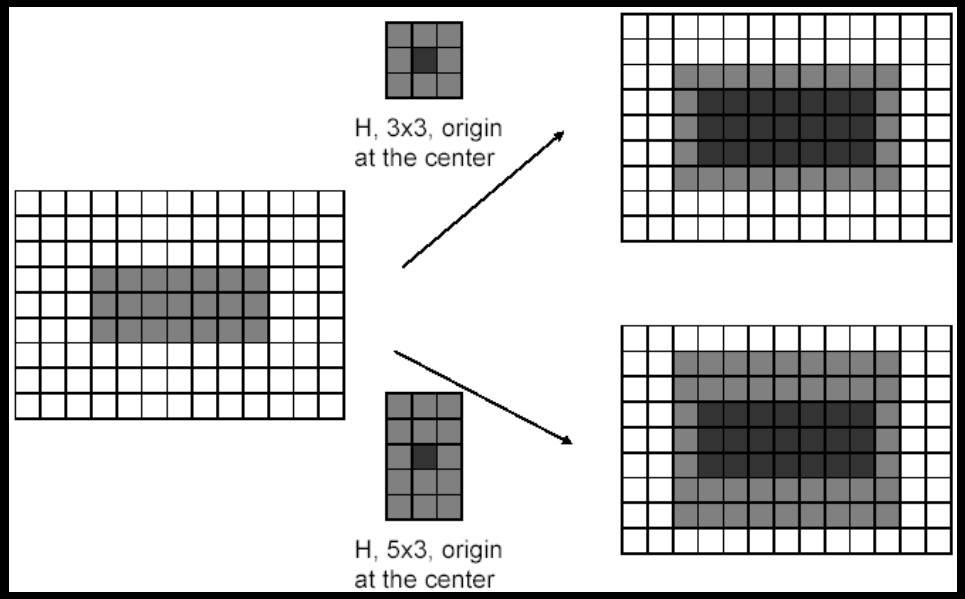


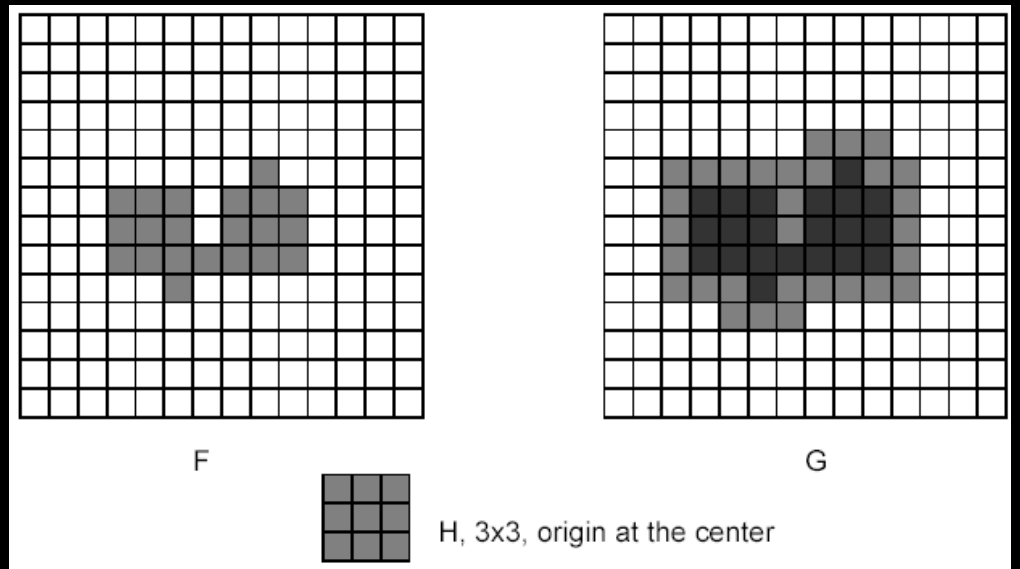


Dilatácia



Dilatácia


H, 3x3, origin
at the center
H, 5x3, origin
at the center


F
G
H, 3x3, origin at the center

Vlastnosti dilatácie

$$A \oplus B = B \oplus A$$

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

$$A_1 \subseteq A_2 \Rightarrow (A_1 \oplus B) \subseteq (A_2 \oplus B)$$

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$

Erózia

Minkowského rozdiel $\ominus$

$$A \ominus B = \bigcap_{b \in B} A_{-b}$$

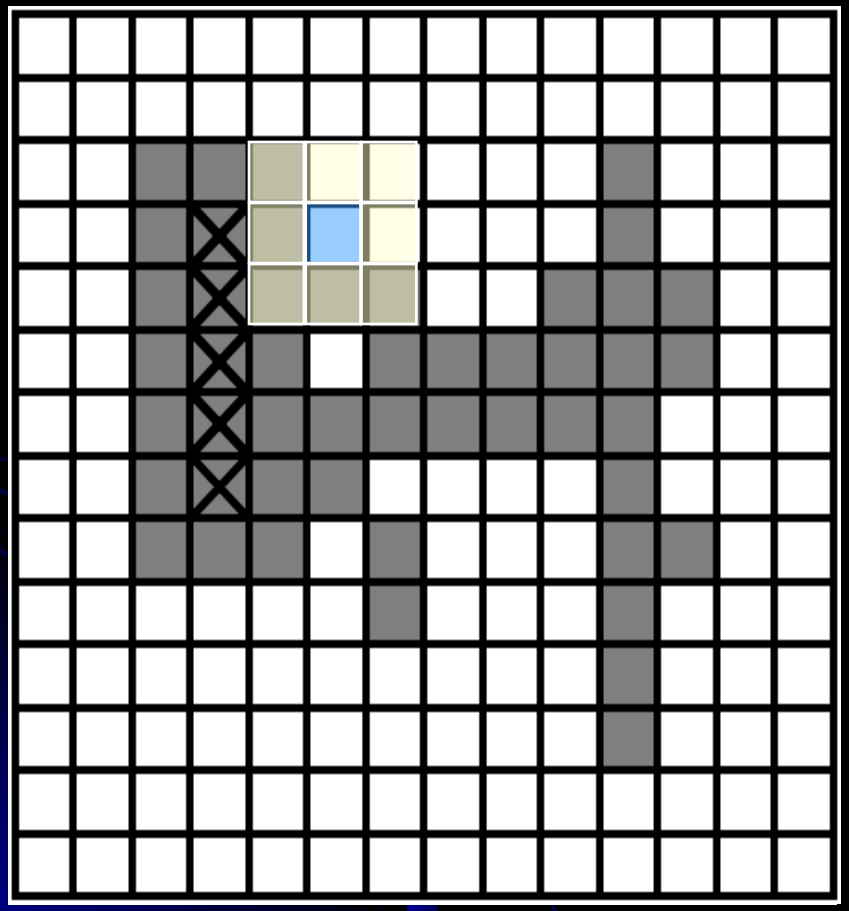
$$A \ominus B = \bigcap_{b \in B} a - b \mid a \in A$$

$$A \ominus B = \quad x \mid B_x \subseteq A$$

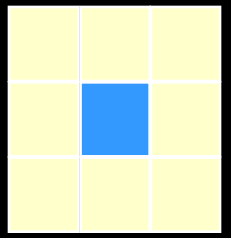
Kontraktívna operácia – zmenšuje množinu

$$A \ominus B = \{ x \mid B_x \subseteq A \}$$

Erózia

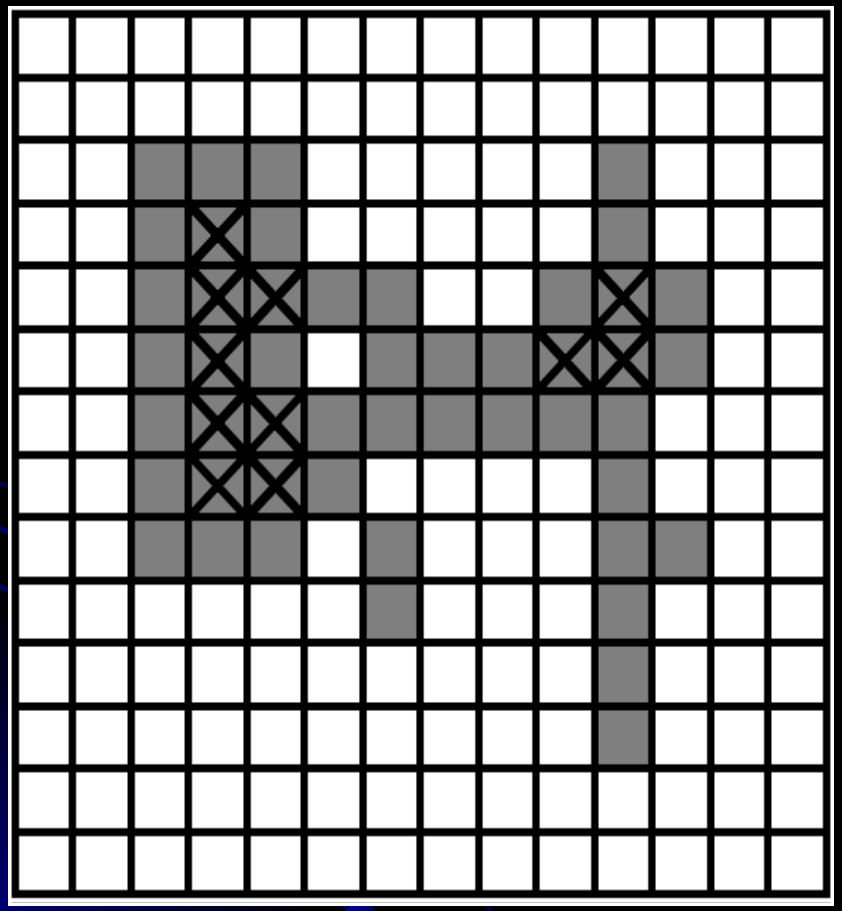


SE=

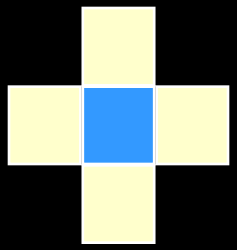


$$A \ominus B = \{ x \mid B_x \subseteq A \}$$

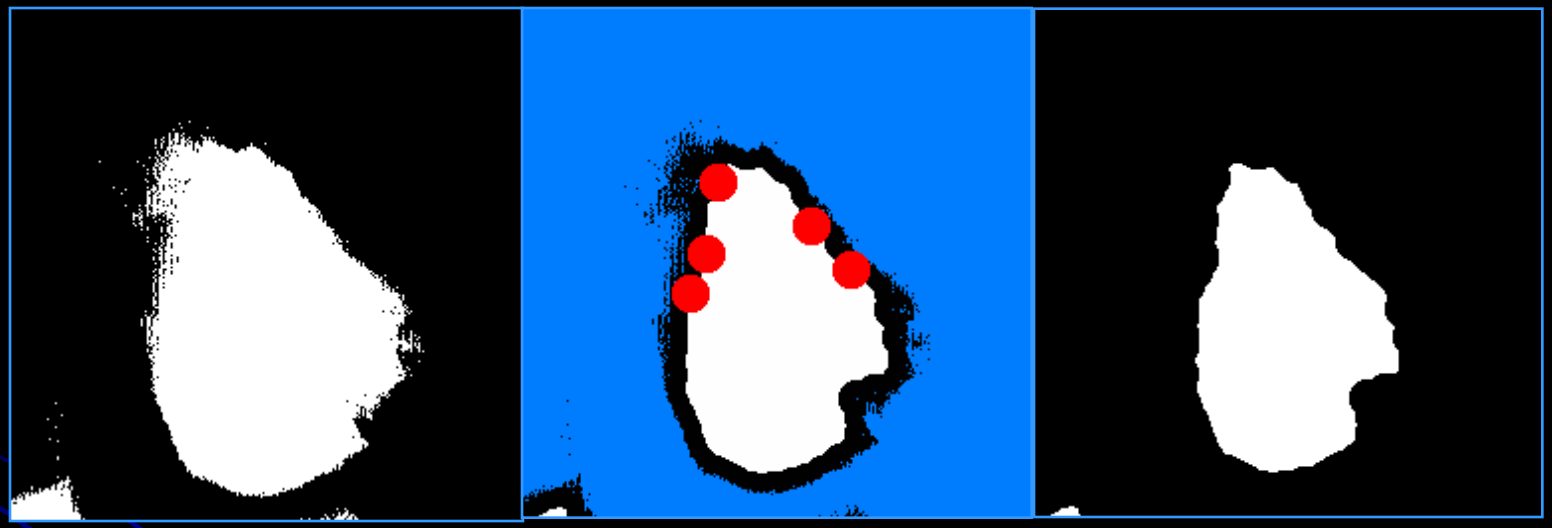
Erózia



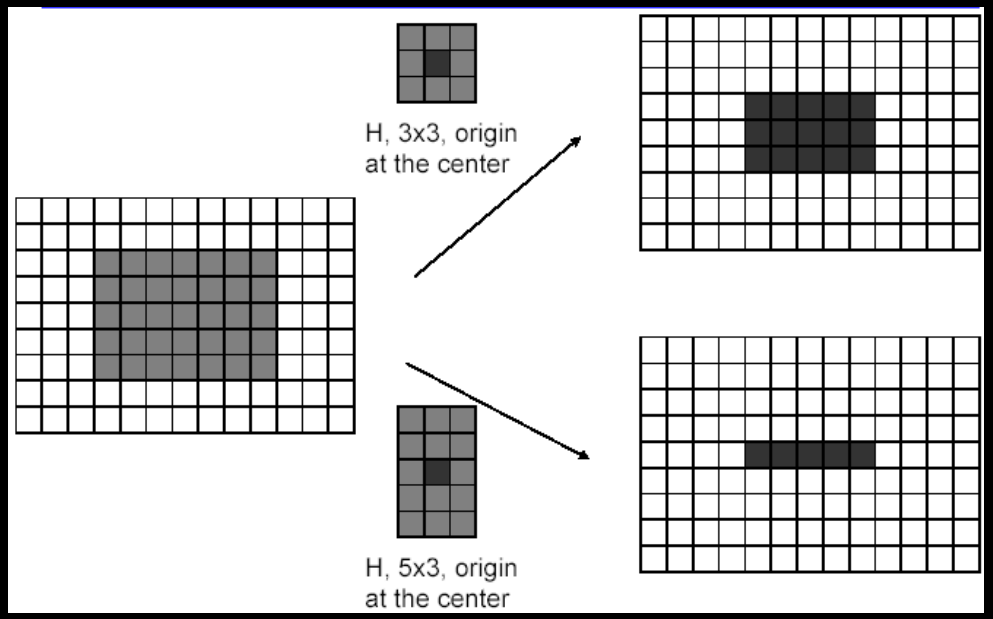
SE=

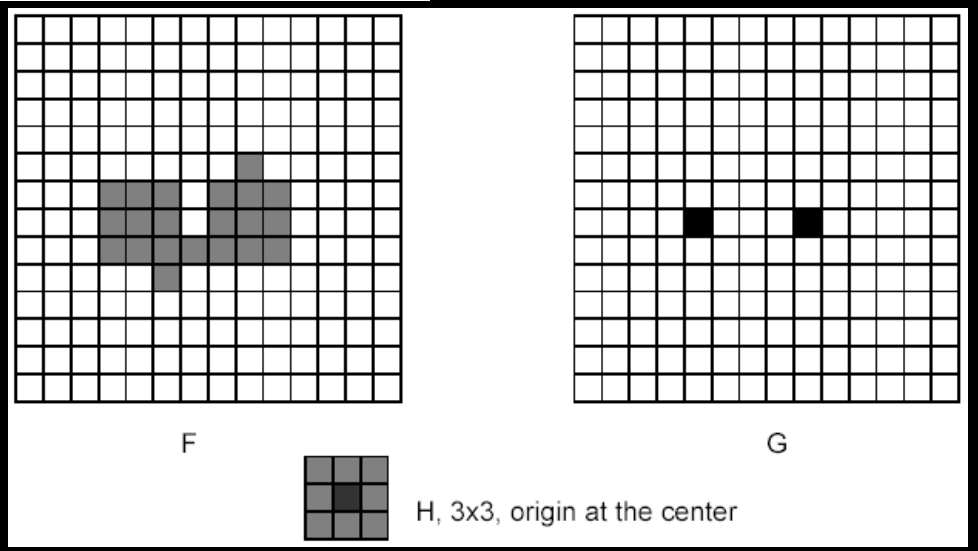


Erózia



Erózia


H, 3x3, origin
at the center
H, 5x3, origin
at the center


F
G
H, 3x3, origin at the center

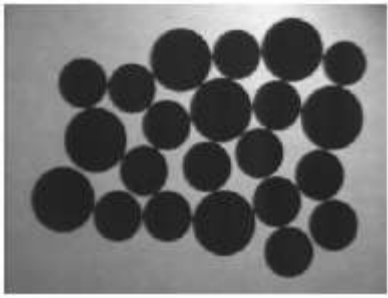
Vlastnosti erózie

$$A \ominus B \neq B \ominus A$$

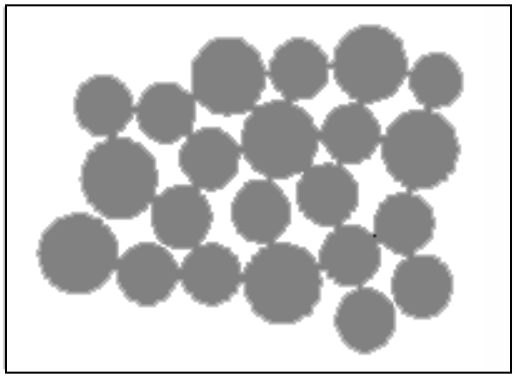
$$A_1 \subseteq A_2 \Rightarrow (A_1 \ominus B) \subseteq (A_2 \ominus B)$$
$$B_1 \subseteq B_2 \Rightarrow (A \ominus B_1) \supseteq (A \ominus B_2)$$
$$A \ominus (B \cup C) = (A \ominus B) \cap (A \ominus C)$$
$$(A \ominus B) \ominus C = A \ominus (B \oplus C)$$

$$(A \ominus B) \oplus B \subseteq A \subseteq (A \oplus B) \ominus B$$

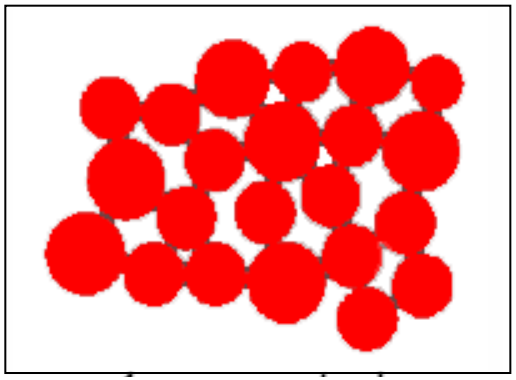
Príklad použitia



Original



Prahovanie



1 spojitý útvar

Erózia

Originál Erodovaný 1x Erodovaný 2x

Dualita

$$A \ominus E^{\tau} = A^{C} \oplus \hat{E}$$

Opakovanie

dilatácia

Zväčšuje množinu

Vypĺňa diery, zálivy určitej veľkosti a tvaru

erózia

Zmenšuje množinu

Odstraňuje štruktúry určitej veľkosti a tvaru

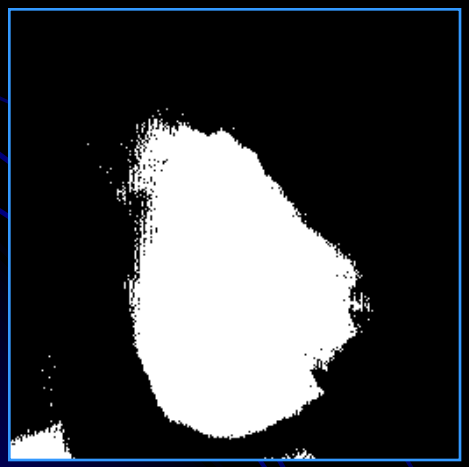
Môže rozdeliť množinu

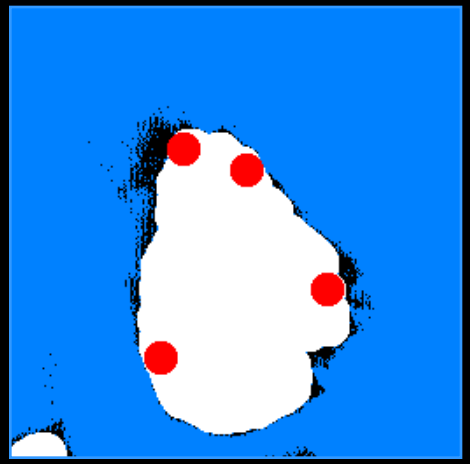
– v závislosti na štrukturálnom prvku

Interaktívne na http://dip.sccg.sk/

Otvorenie

$$A \circ B = (A \ominus B) \oplus B$$





Vlastnosti otvorenia

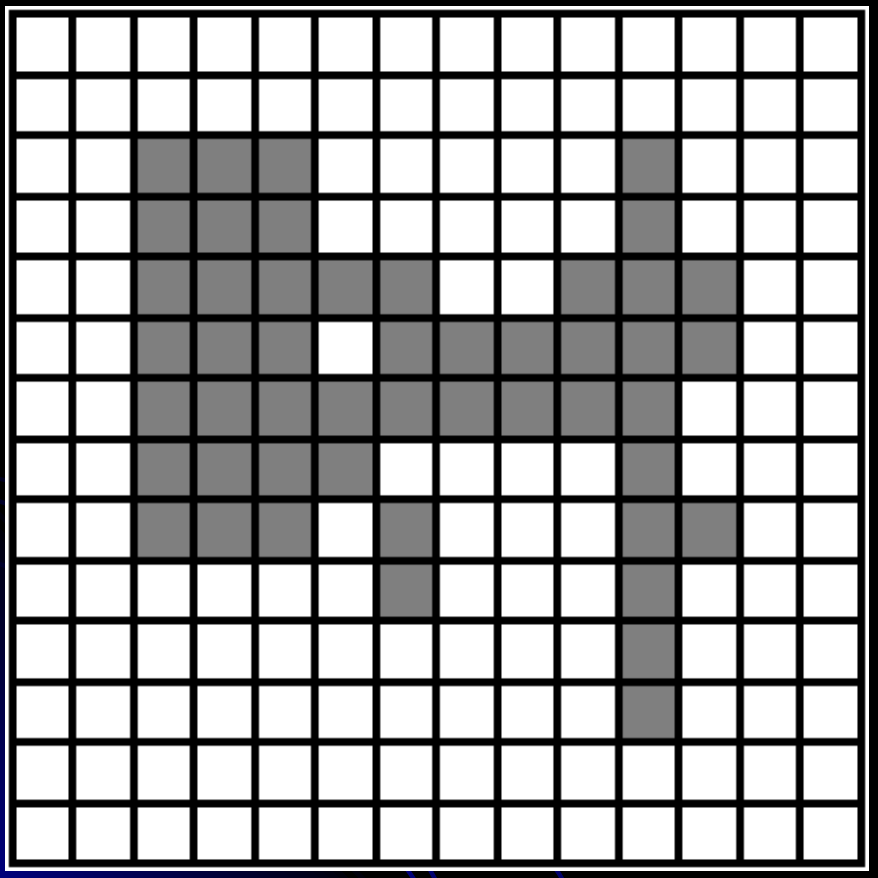
$$A \circ B \subseteq A$$

$$A \circ B \circ B = A \circ B$$

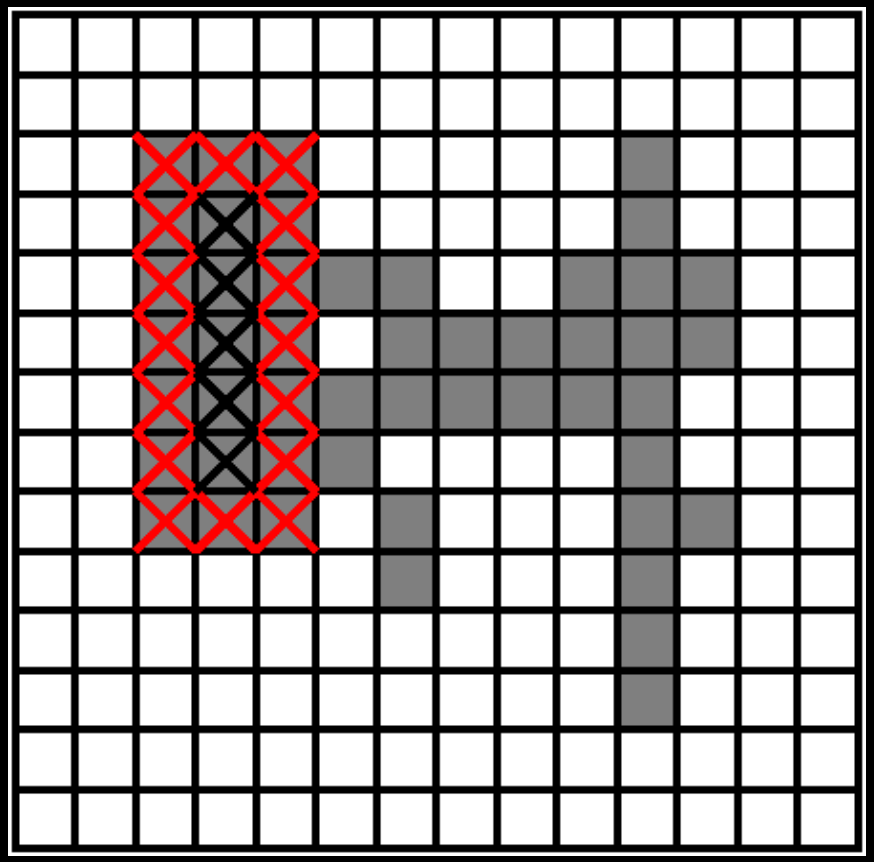
$A \circ B = (A \ominus B) \oplus B$

Otvorenie

B=



A

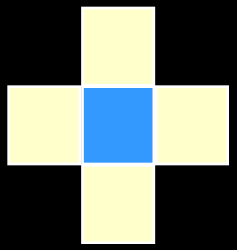


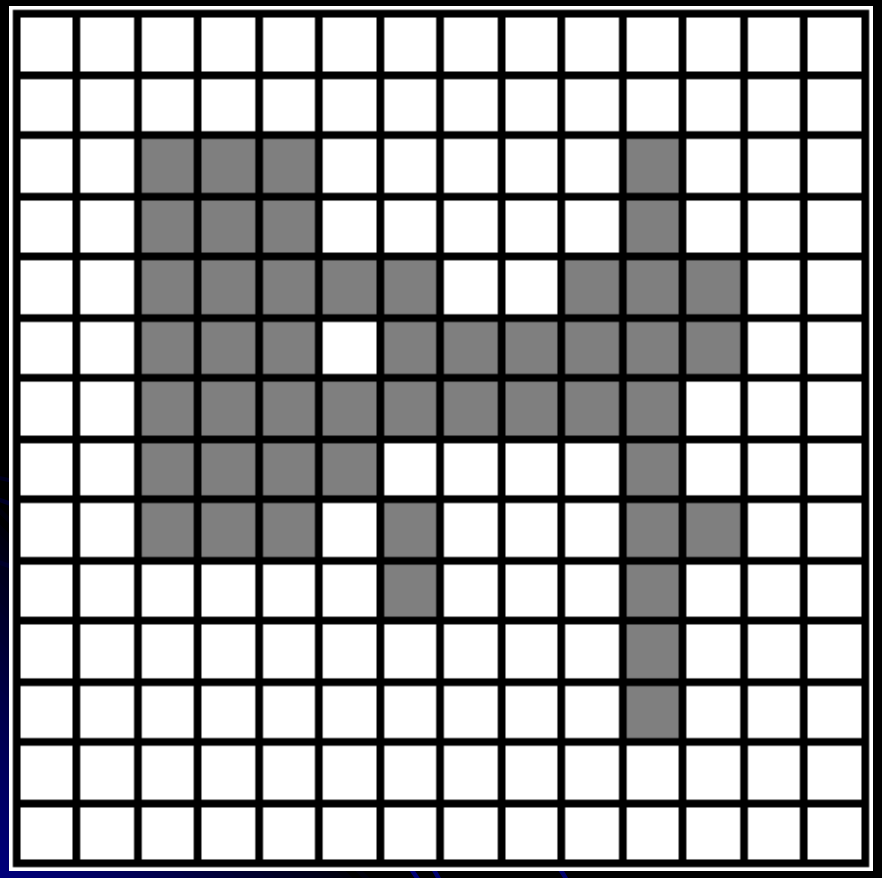
$A \ominus B$ $A \circ B$

$A \circ B = (A \ominus B) \oplus B$

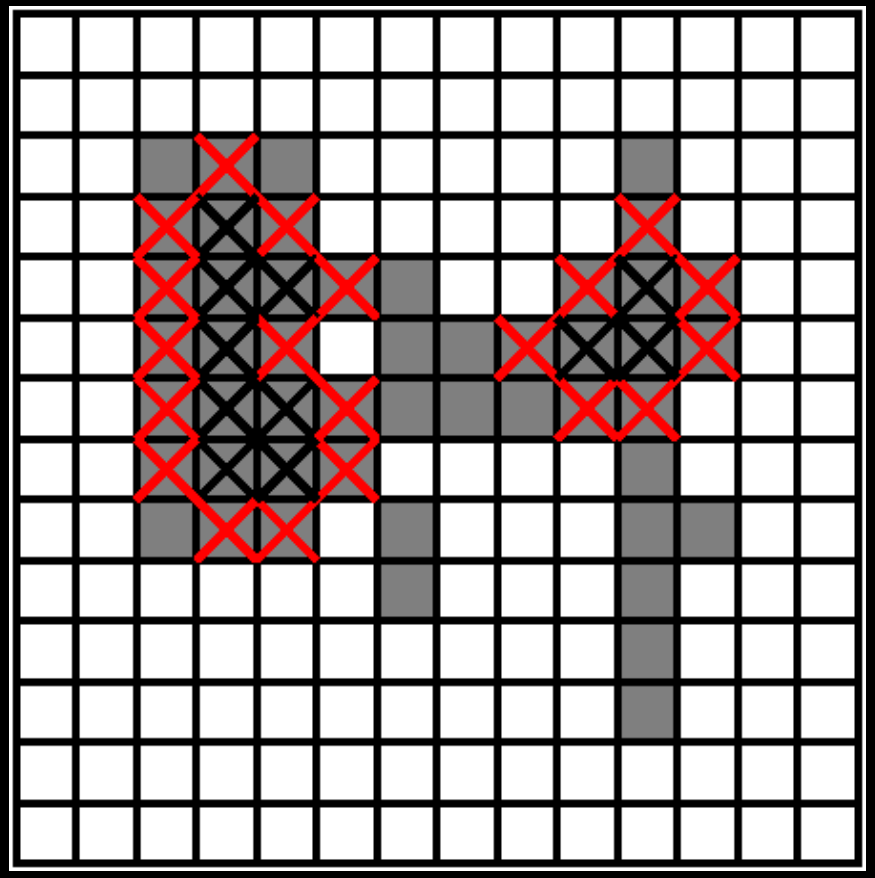
Otvorenie

B=



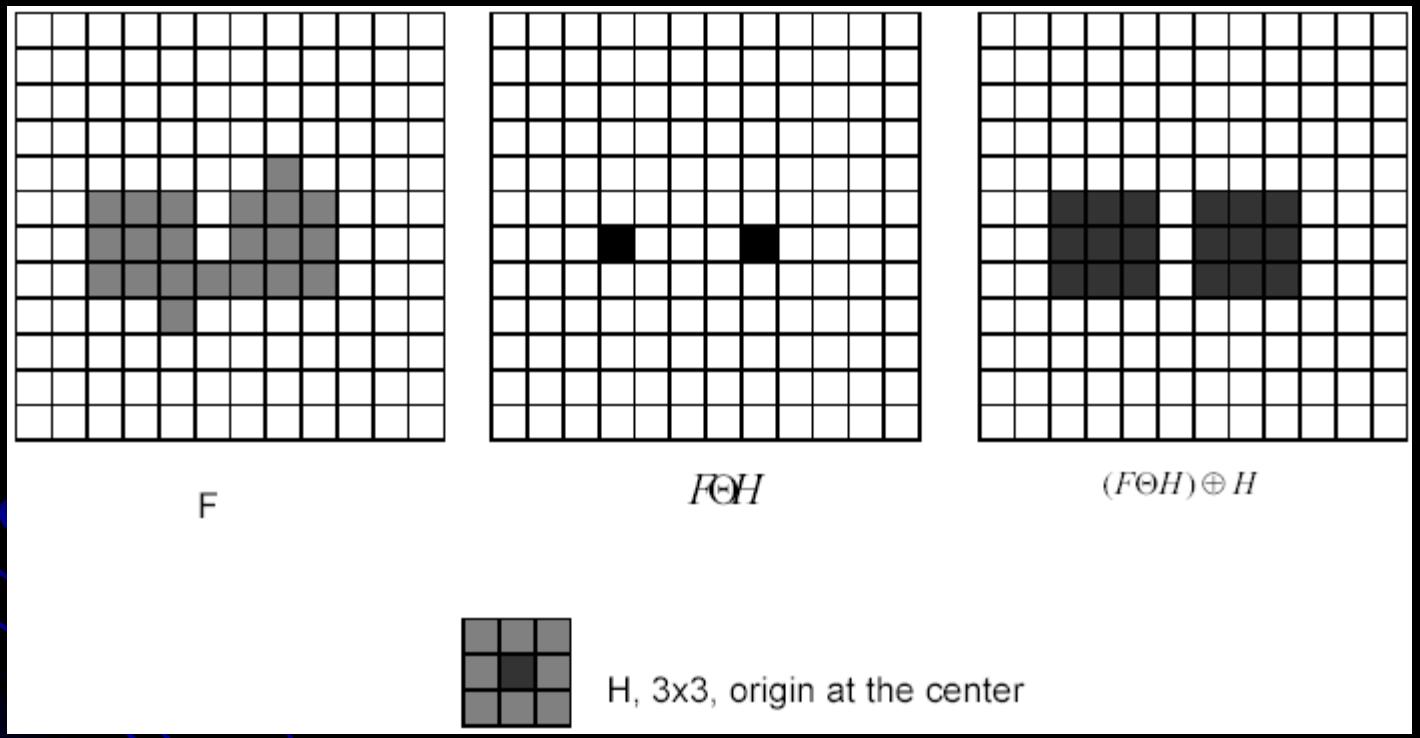


A



$A \ominus B$ $A \circ B$

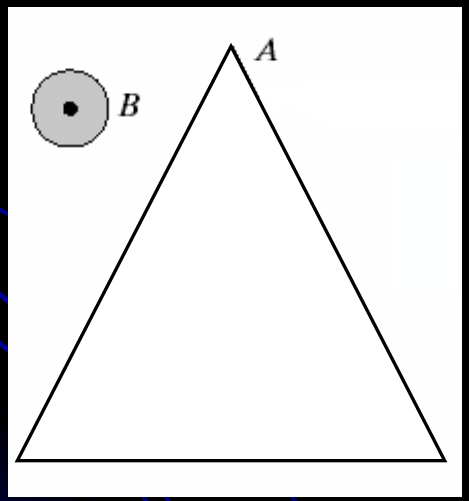
Otvorenie



Otvorenie

$$A \circ B = \bigcup B_x \mid B_x \subseteq A$$

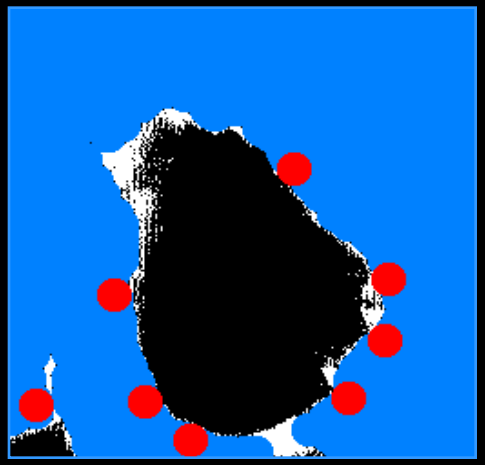
posúvame B **po vnútornej strane hranice** A



$A \ominus B$ $\oplus B$ $A \circ B$

Uzavretie

$$A \bullet B = (A \oplus B) \ominus B$$



Vlastnosti uzavretia

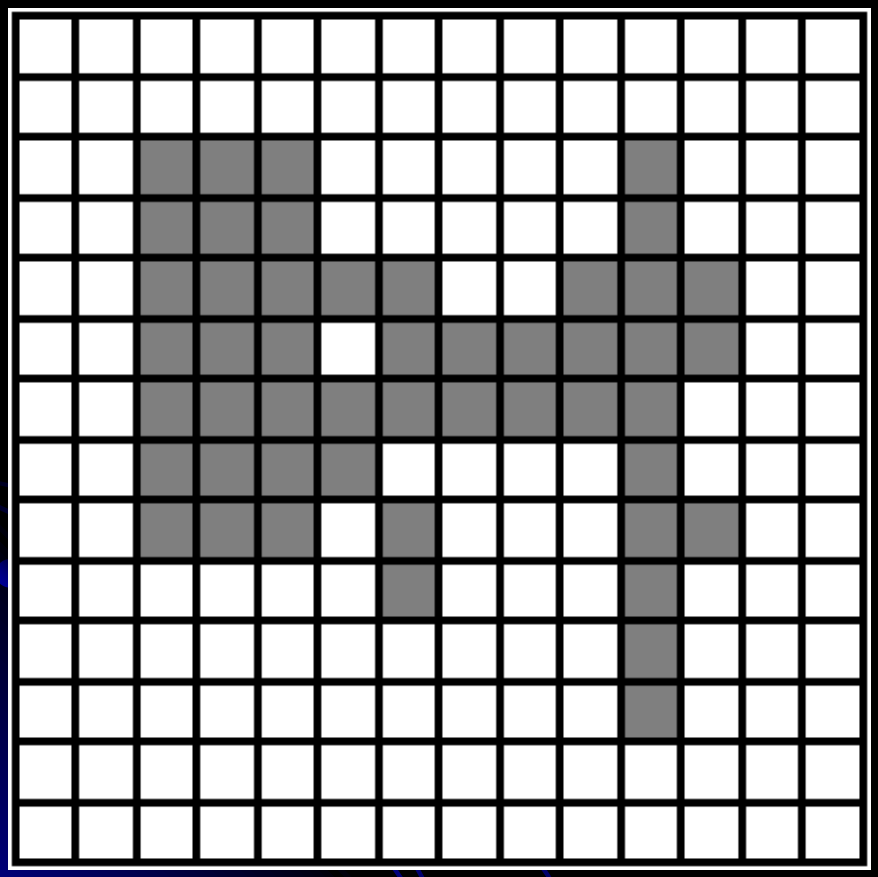
$$A \subseteq A \bullet B$$

$$\overline{A \bullet B} \bullet B = A \bullet B$$

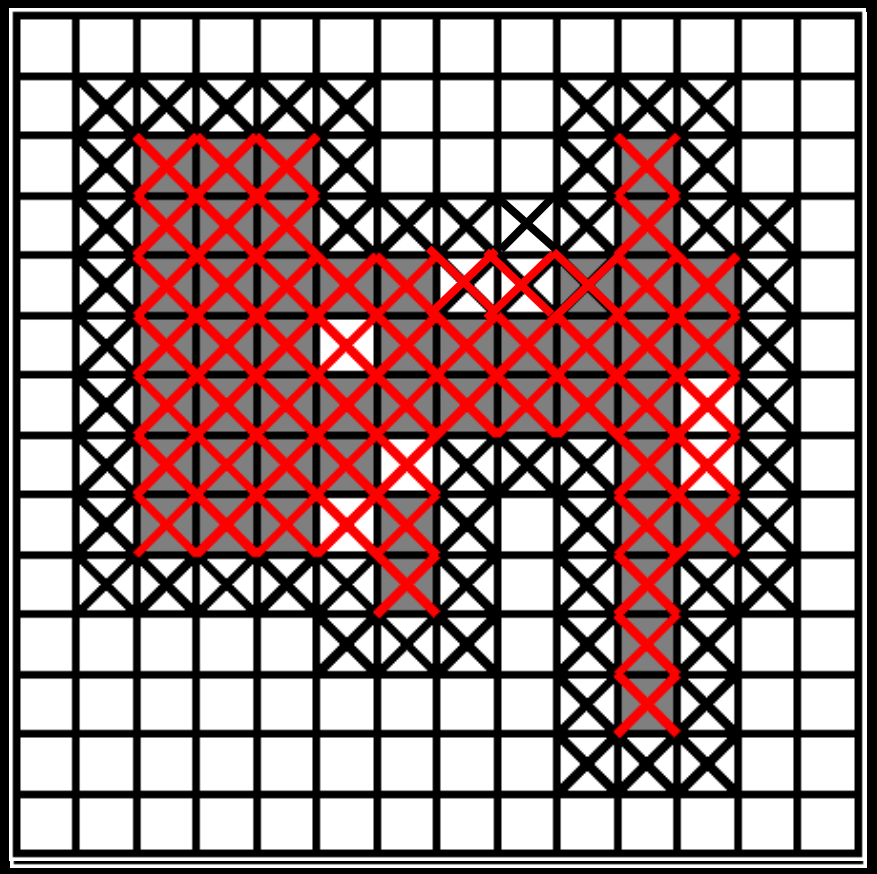
$A \bullet B = (A \oplus B) \ominus B$

Uzavretie

B=



A

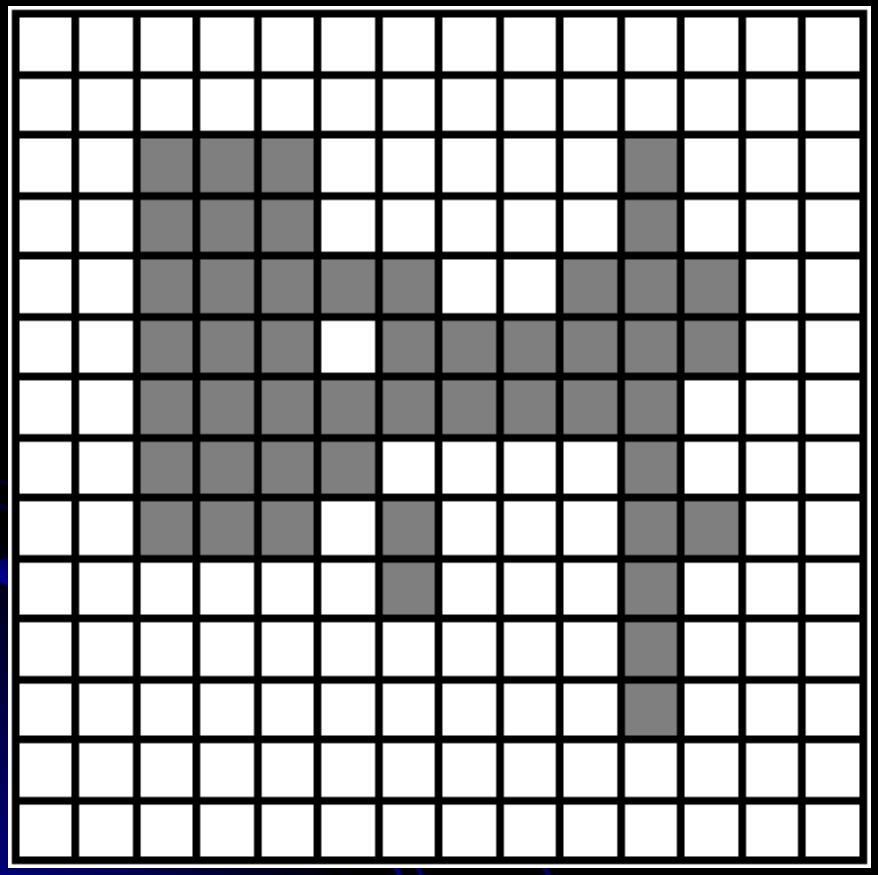


A⊕B A • B

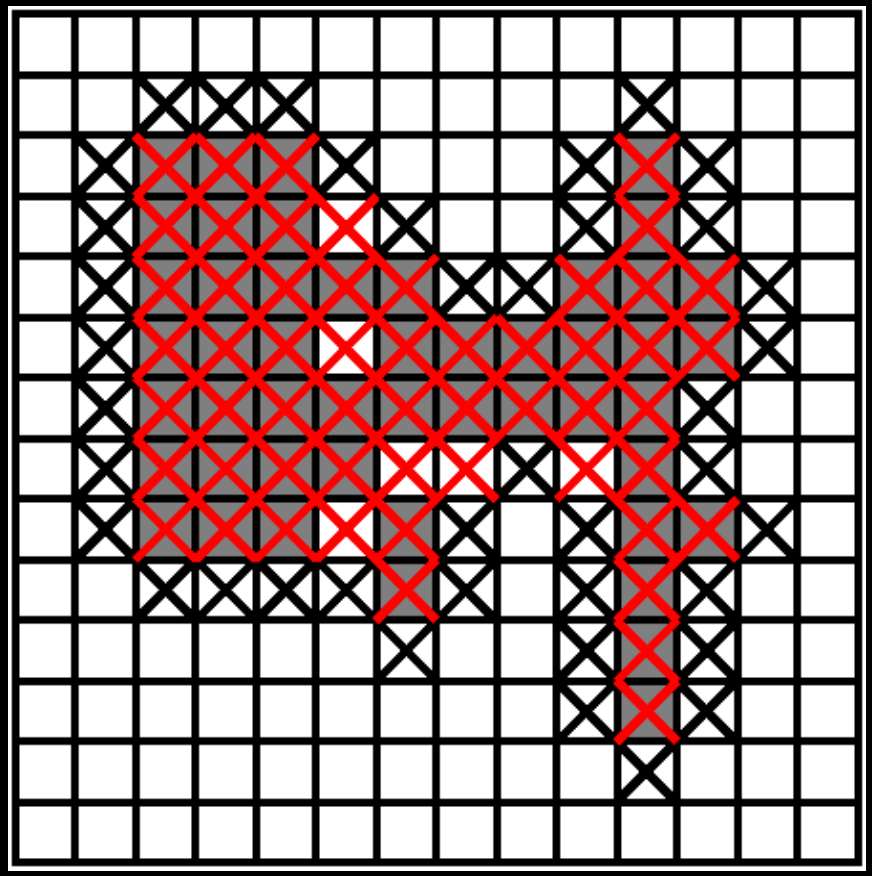
Uzavretie

$A \bullet B = (A \oplus B) \ominus B$

B=

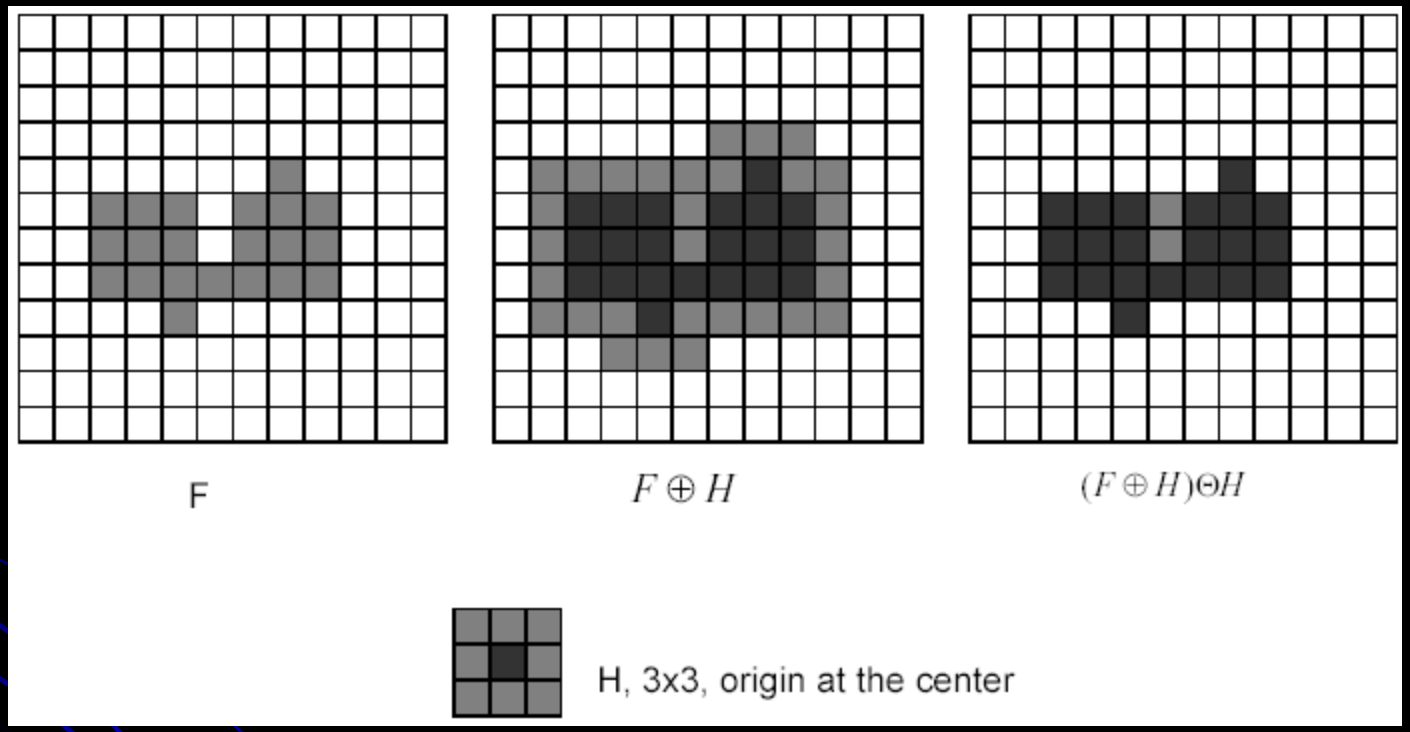


A



A⊕B A • B

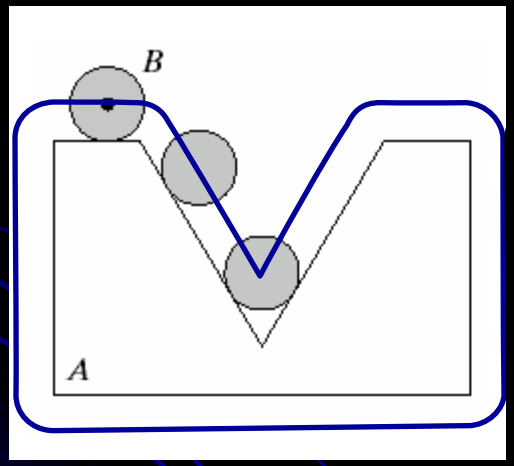
Uzavretie



Uzavretie

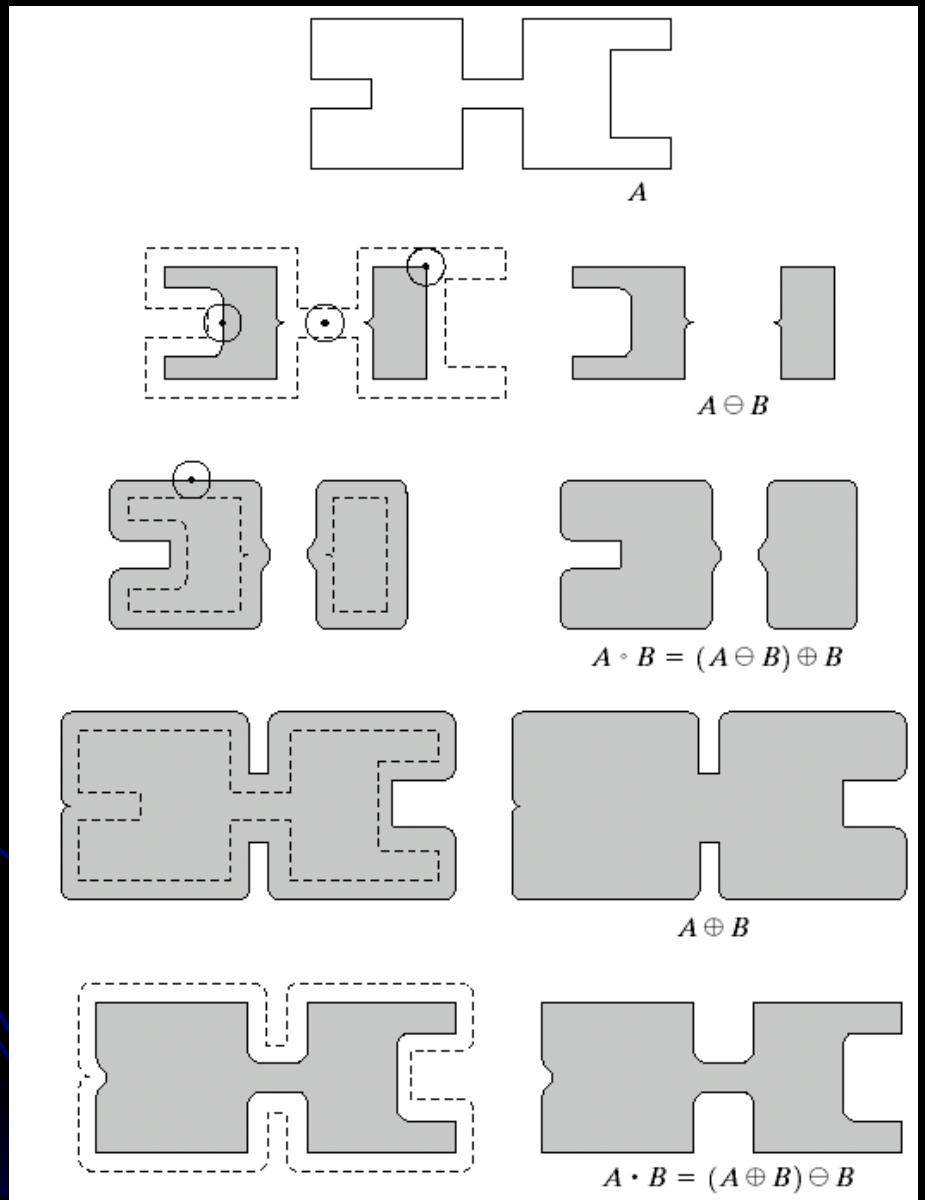
$$A \bullet B = \{ w \mid w \in B_x, B_x \cap A \neq \emptyset \}$$

posúvame B **po vonkajšej strane hranice** A

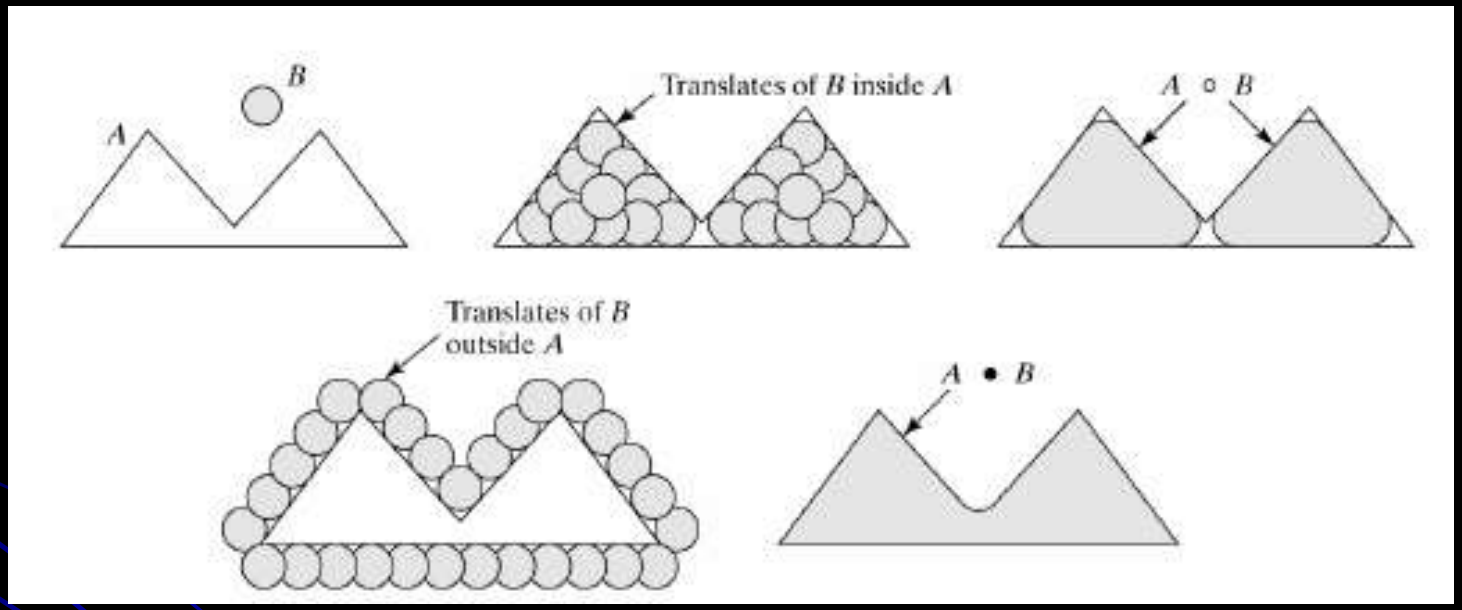


$A \oplus B$ $\ominus B$ $A \bullet B$

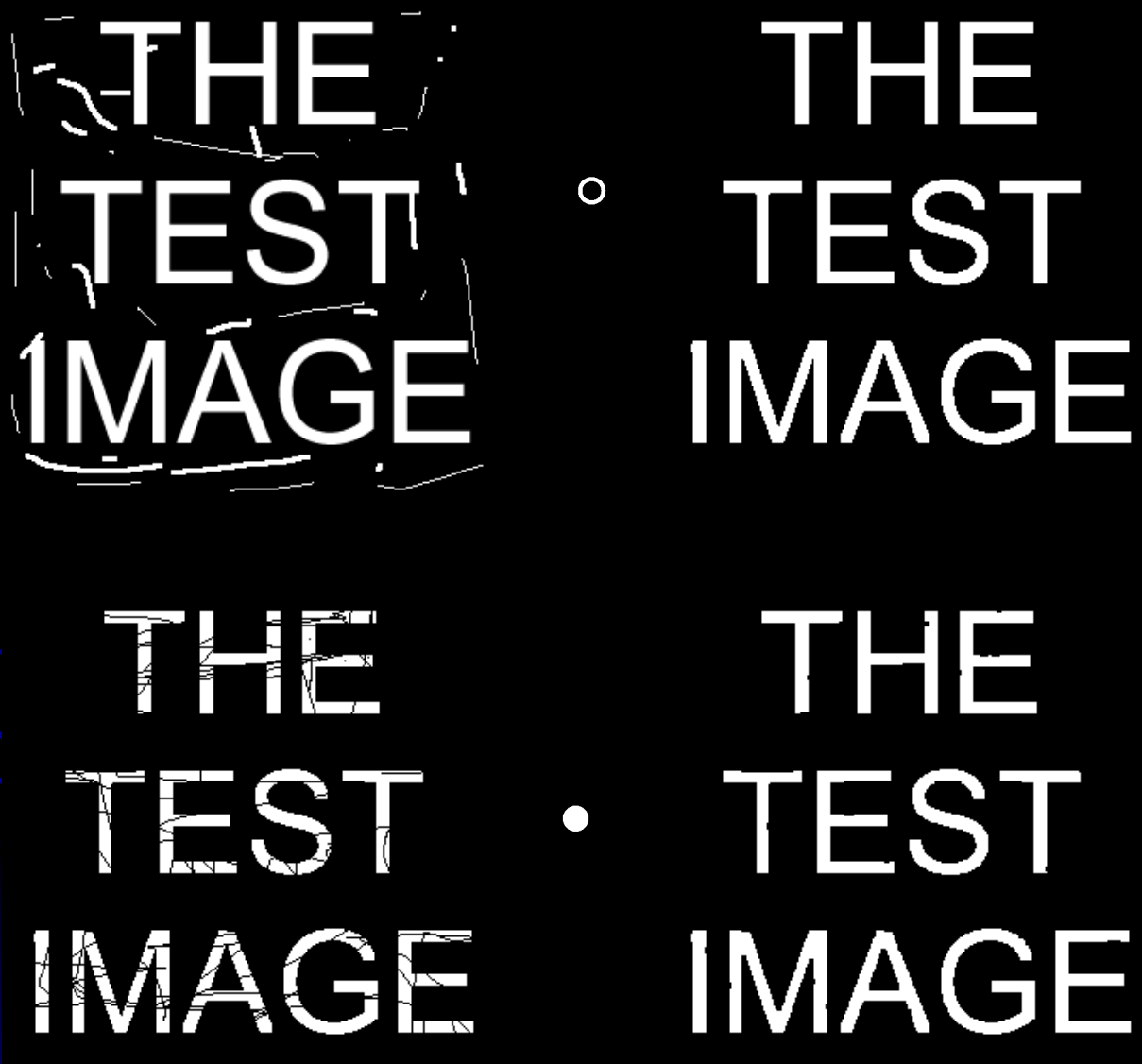
Otvorenie - Uzavretie



Otvorenie - Uzavretie



Otvorenie - Uzavretie



Dualita

$$A \bullet E^{C} = A^{C} \circ \hat{E}$$

Aplikácia: filtrovanie šumu

1. erózia
$A \ominus B$

2. dilatácia
$(A \ominus B) \oplus B =$
$A \circ B$

3. dilatácia
$(A \circ B) \oplus B$

4. erózia
$((A \circ B) \oplus B) \ominus B =$
$(A \circ B) \bullet B$

Opakovanie

otvorenie

erózia + dilatácia

- vyhladzuje kontúry
- prerušuje tenké spojenia
- maže tenké výčnelky

uzavretie

dilatácia + erózia

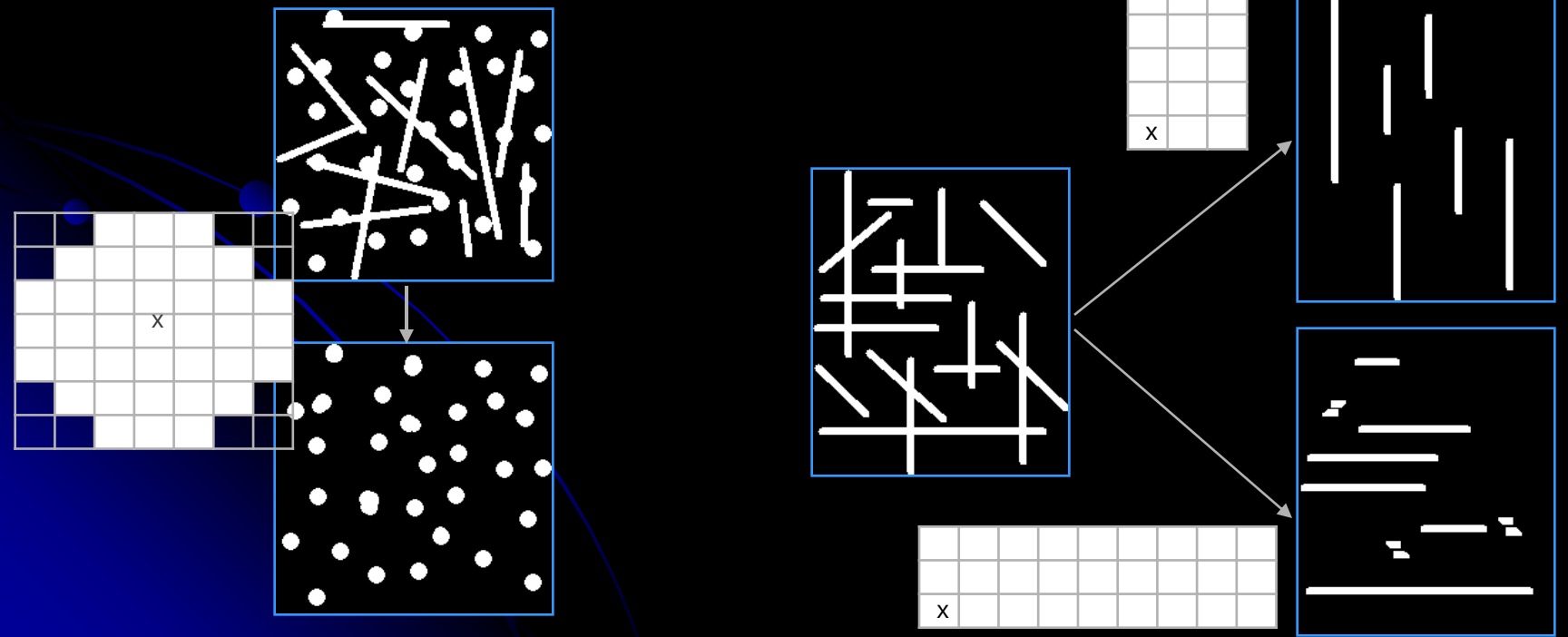
- vyhladzuje kontúry
- spája blízke oblasti
- vypĺňa malé diery a tenké zálivy

Zachovávajú (približnú) veľkosť množiny

Detekcia tvarov

Otvorenie použitím daného štrukturálneho prvku

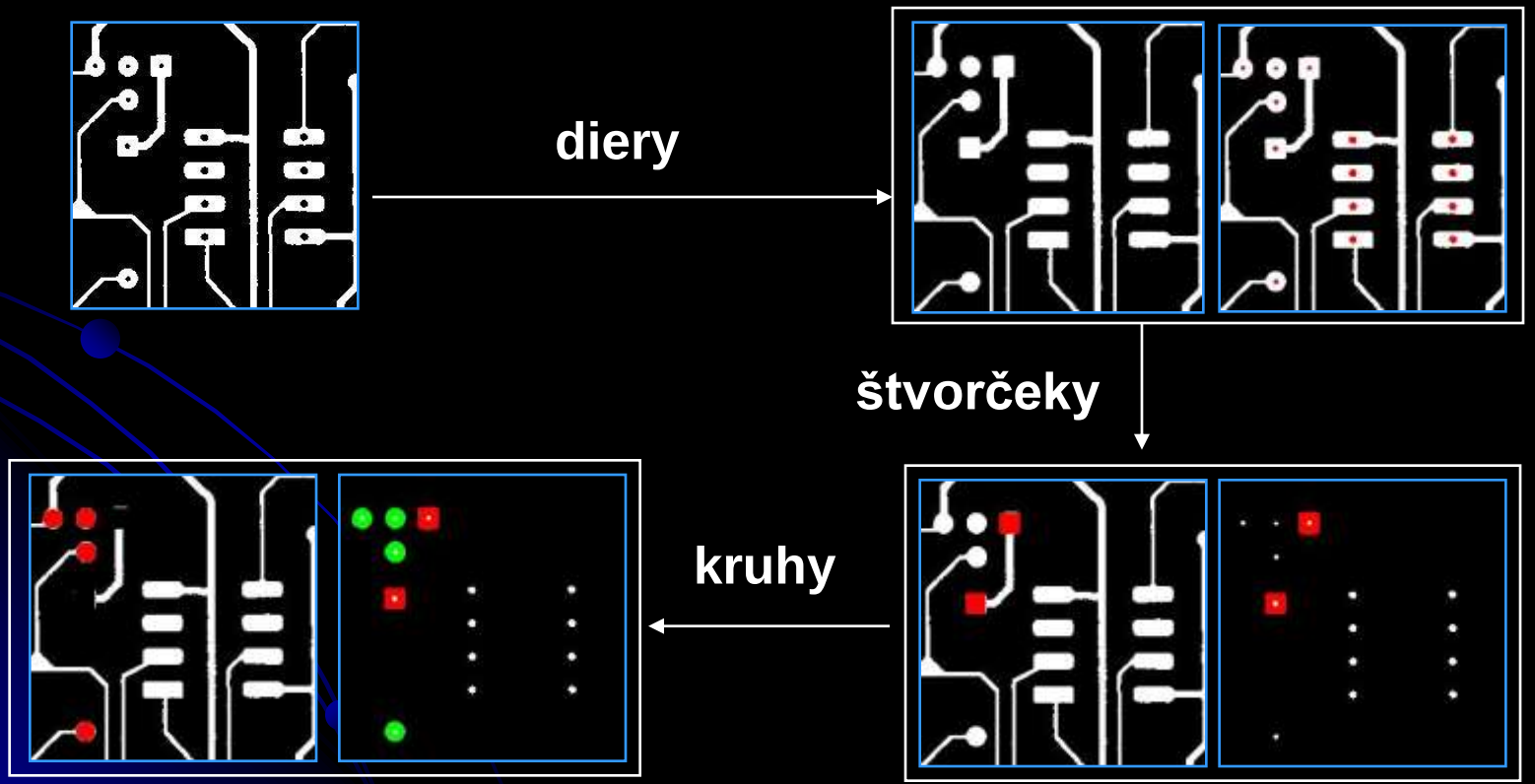
Príklad 1: detekcia kruhov, čiar



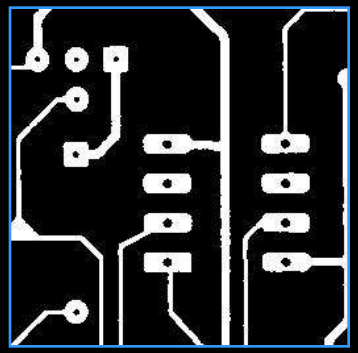
Detekcia tvarov

Príklad 2:

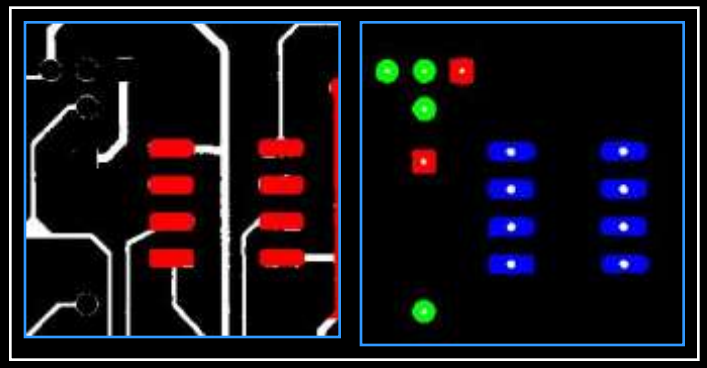
detekcia jednotlivých prvkov plošného spoja

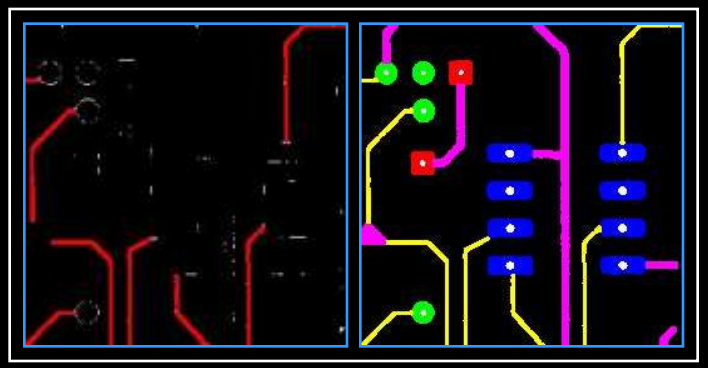


Detekcia tvarov

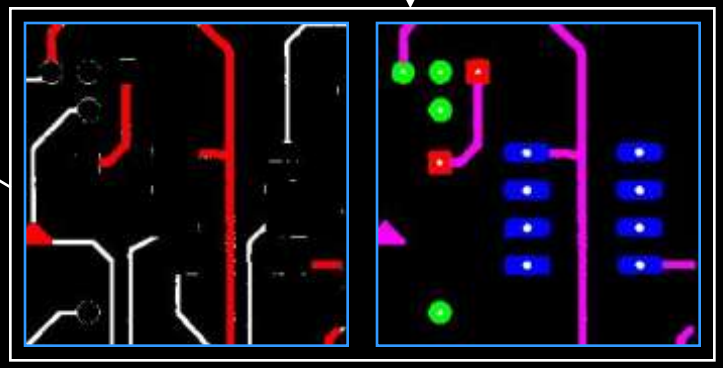


obdĺžniky



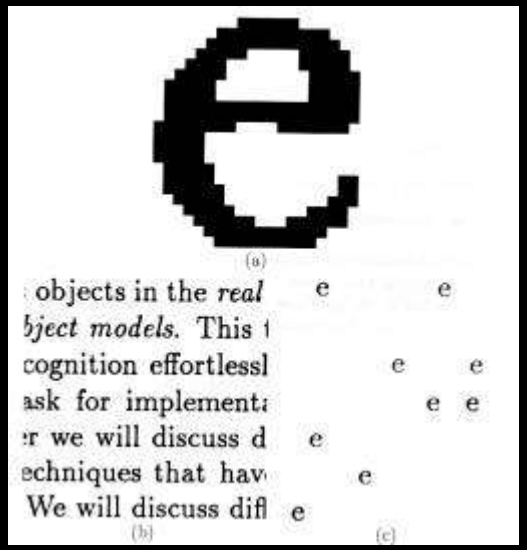


hrubé spoje

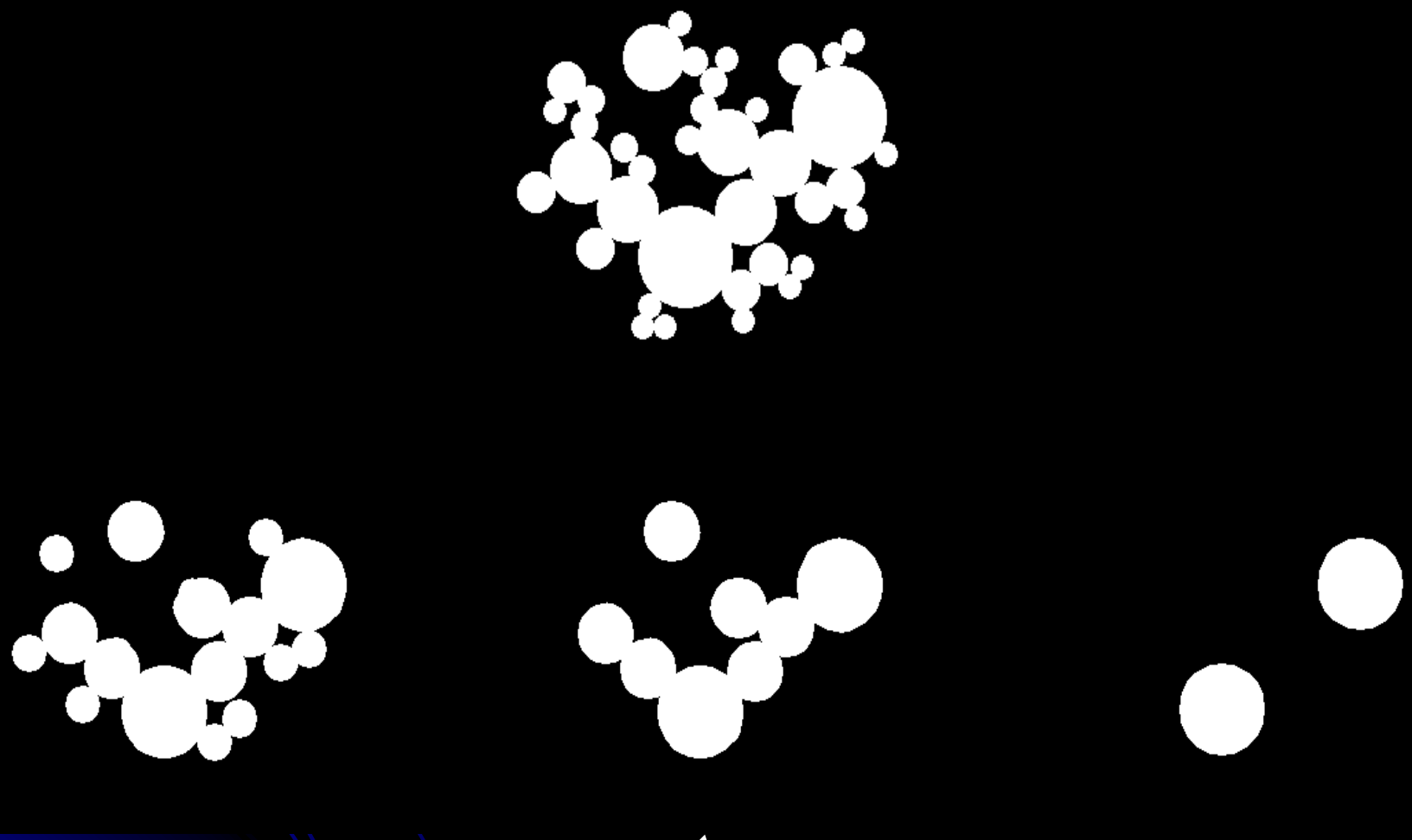


tenké spoje

Text


(a)
objects in the *real*
bject models. This
cognition effortlessl
ask for implementa
r we will discuss d
echniques that hav
We will discuss dif
(b)
e
e
e
e
e
e
e
e
(c)

Granulometria

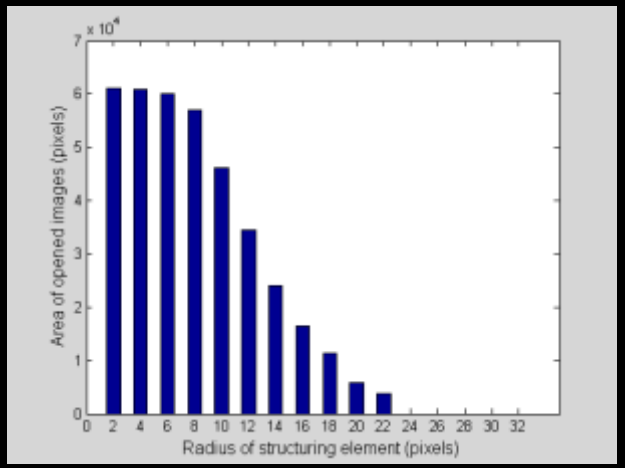


Otvorenie – kruh s priemerom 10, 15 a 25

Granulometria

Otvorenie so zväčšujúcim sa SE:

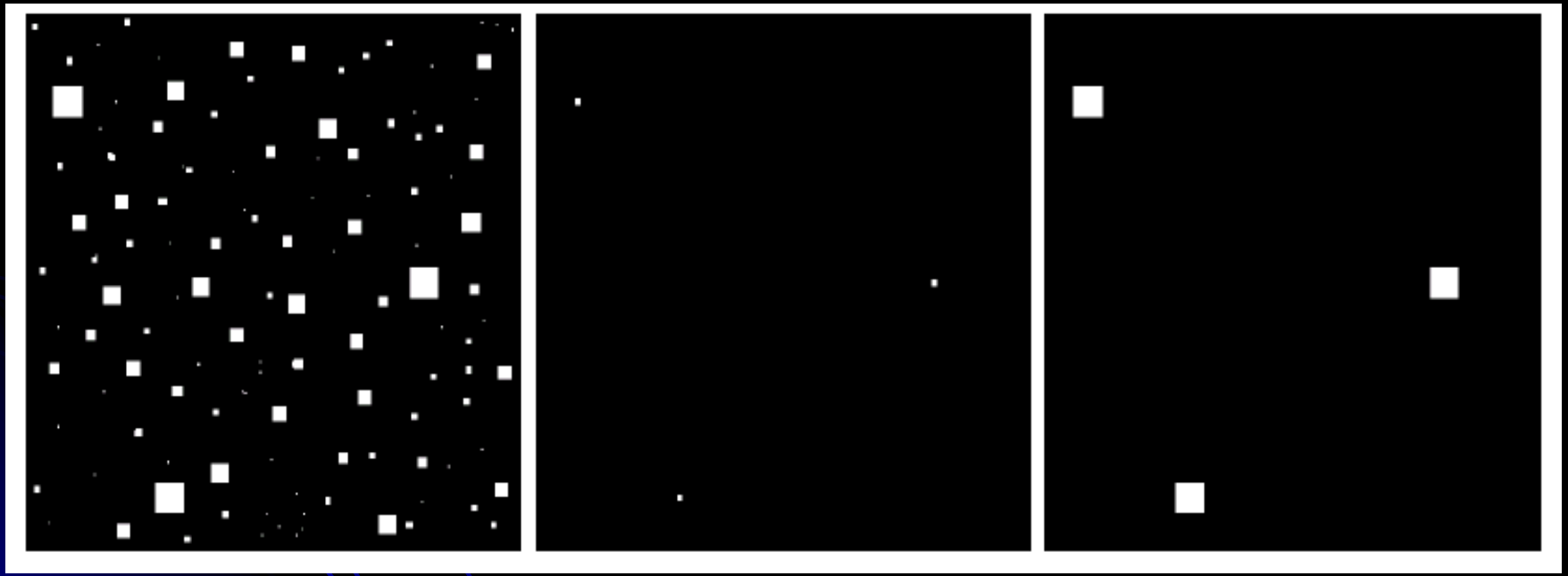
Distribúcia veľkosti granúl



vstup:
štvorce veľkosti
1x1, 3x3, 5x5, 7x7,
9x9 a 15x15 pixlov

erózia:
ŠP 13x13

dilatácia:
ŠP 13x13



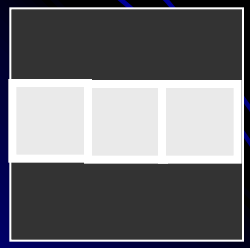
Hit-and-Miss

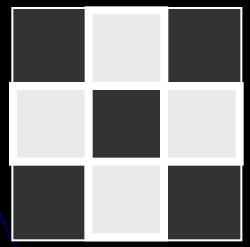
detektor tvarov

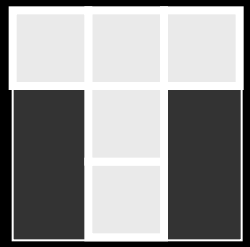
$$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2),$$
$$B_1 \cap B_2 = \varnothing,\ B_1 \cup B_2 = B$$

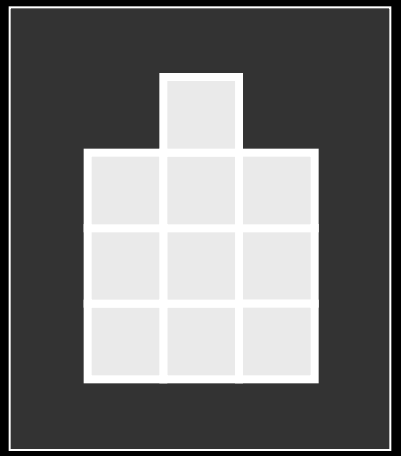
$$A \otimes B = A \ominus B_1 - A \oplus \hat{B}_2$$

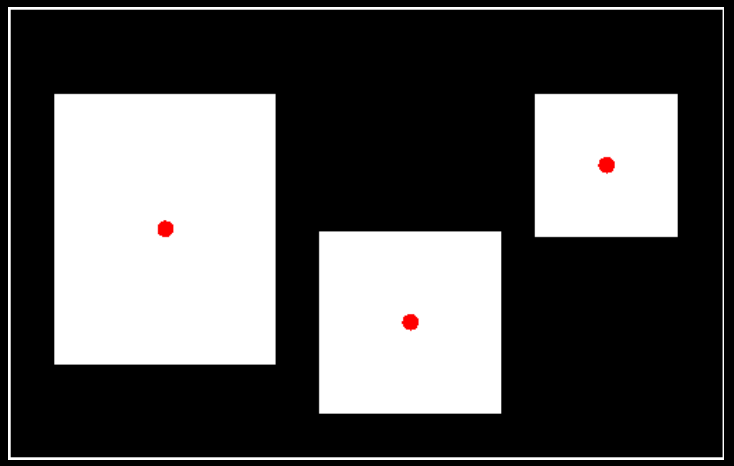
”template matching”



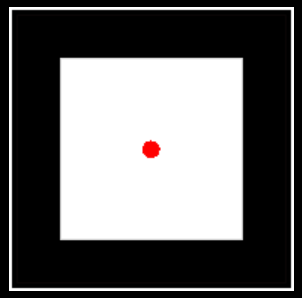




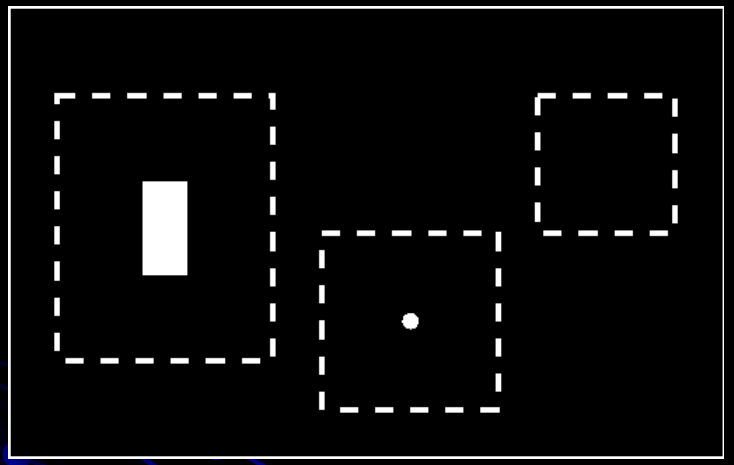




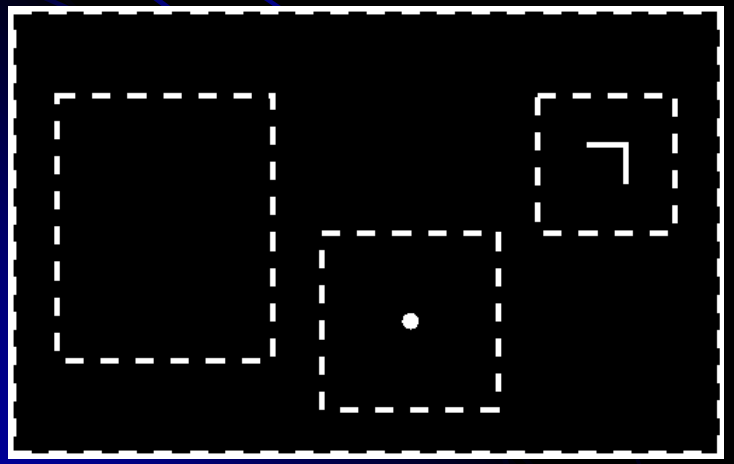
$A=X \cup Y \cup Z$



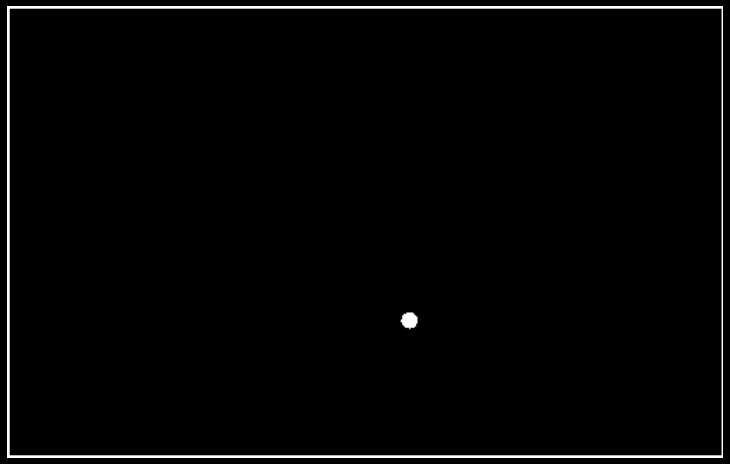
$B_1 \cup B_2$



$A \ominus B_1$

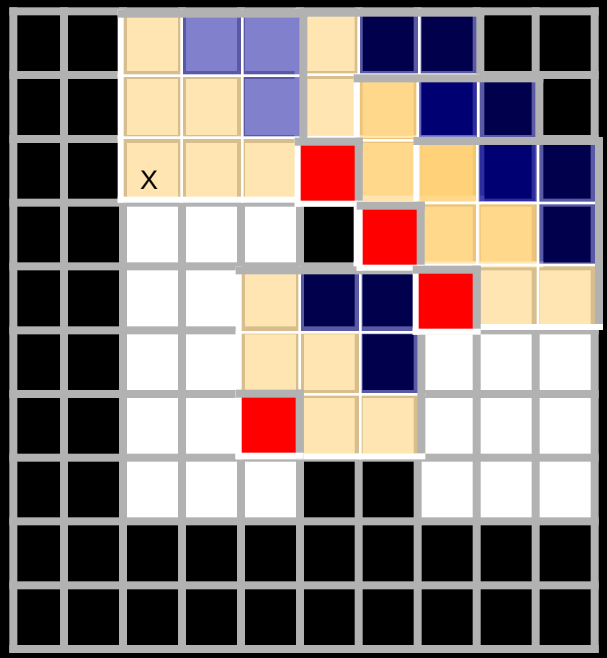


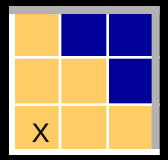
$A^c \ominus B_2$



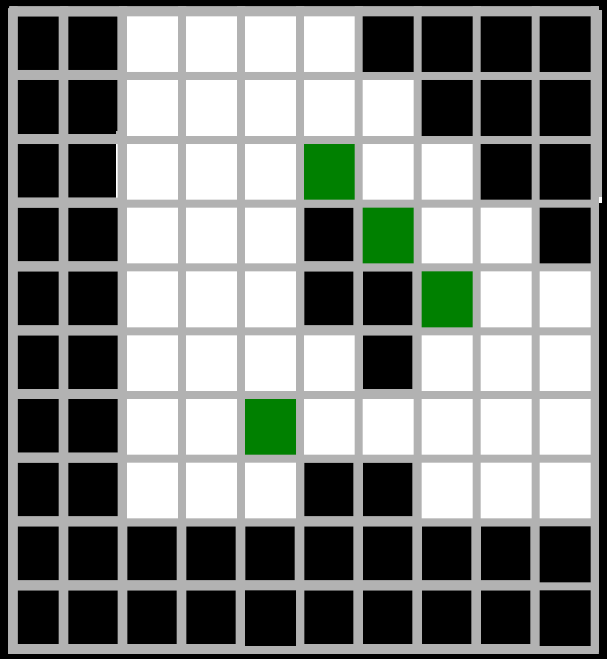
$A \otimes X=(A \ominus B_1) \cap (A^c \ominus B_2)$

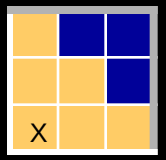
Hit-and-Miss



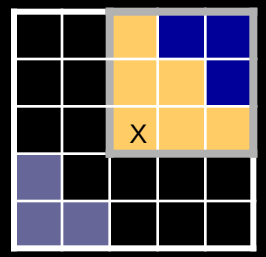


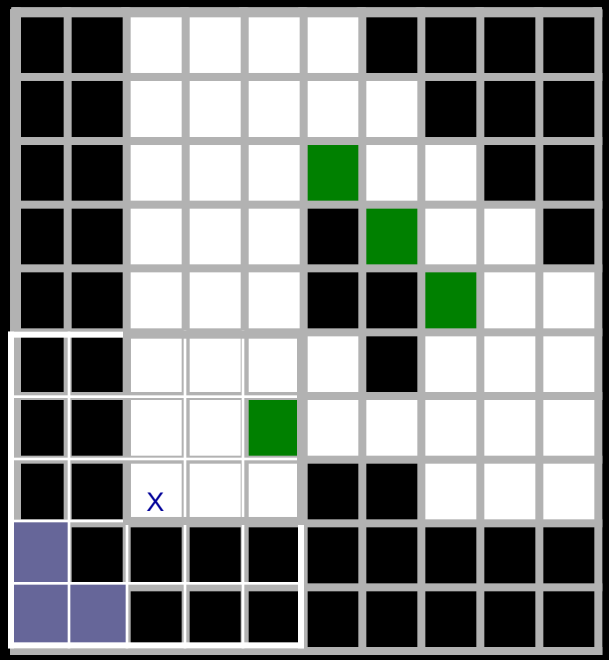
Hit-and-Miss





Hit-and-Miss

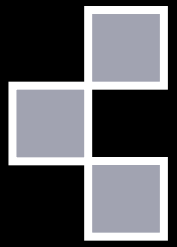




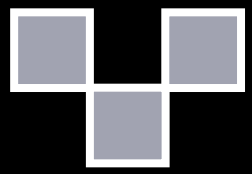
Hit-and-Miss

koncové body

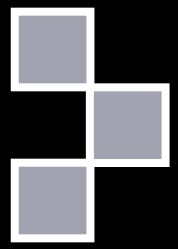
B_1



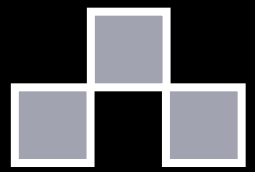
B_2



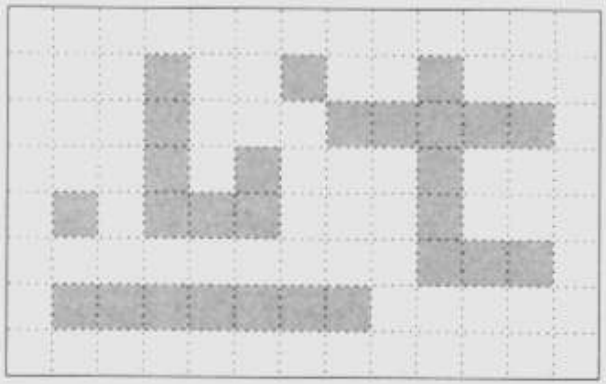
B_2



B_2

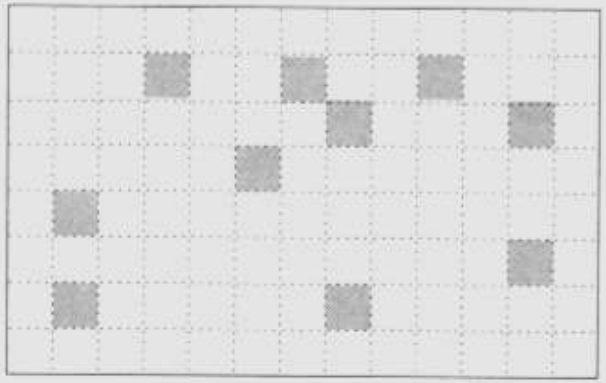


B_2



(a) Input image

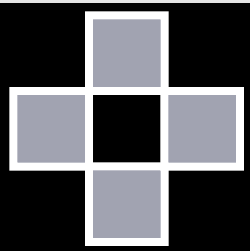
(b) SEs for 4-connected endpoints



(c) Endpoints of input image

izolované body

B_1

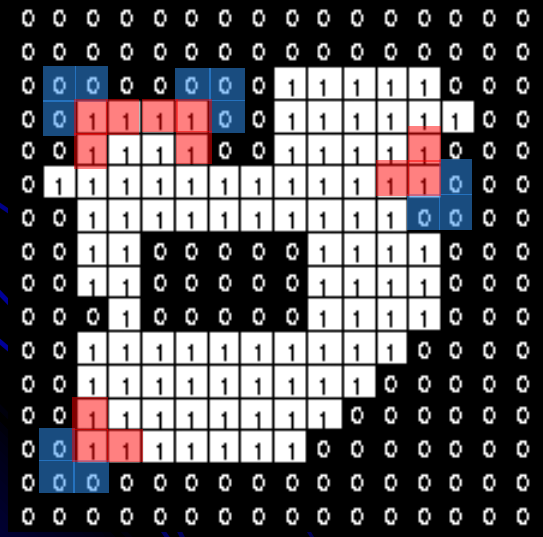


B_2

Hit-and-Miss

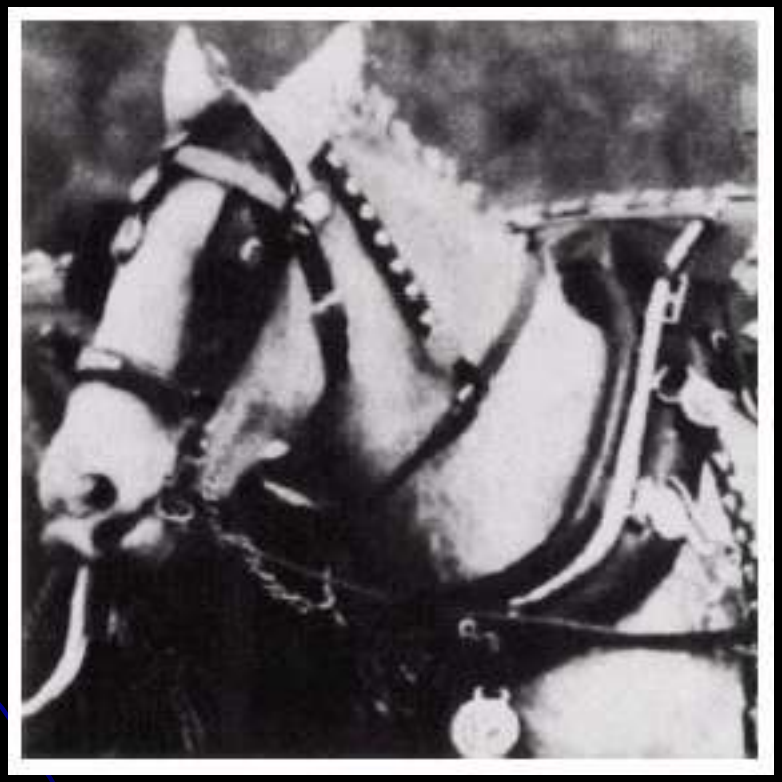
Detekcia rohov

	1				1				0	0		0	0		
0	1	1		1	1	0		1	1	0		0	1	1	
0	0				0	0			1				1		



Konvexné rohy

Šedotónová morfológia



Obraz

$X = \{ (\boldsymbol{a}; f_X(\boldsymbol{a})) \mid \boldsymbol{a} \in E^{n-1}, f_X(\boldsymbol{a}) \in R \cup \{\infty\} \cup \{-\infty\} \}$

= n-dim graf

Nosič (support): $supp(X) = \{ \boldsymbol{a} \in E^{n-1}, f(\boldsymbol{a}) \in R \}$

mimo: ∞ alebo $-\infty$

Pre nás n=3

Obraz

Binárny obraz
$f(\boldsymbol{x})$ = konštanta

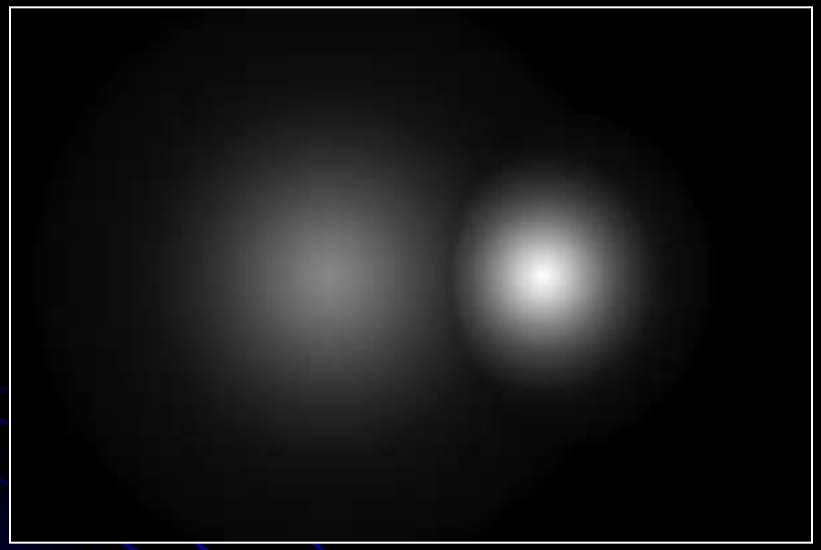
Šedotónový obraz
$f(\boldsymbol{x})$ – úrovne intenzity

$X \subseteq Y$:
$\text{supp}(X) \subseteq \text{supp}(Y)$
$f_X(\boldsymbol{a}) \leq f_Y(\boldsymbol{a})$ pre $\boldsymbol{a} \in \text{supp}(X)$

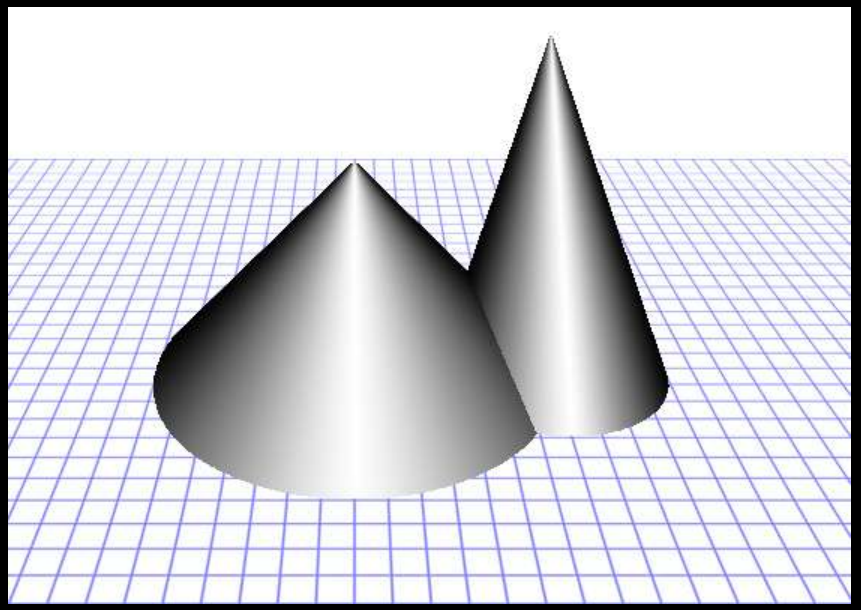
Množinové operácie

min/max
sup/inf

Obraz

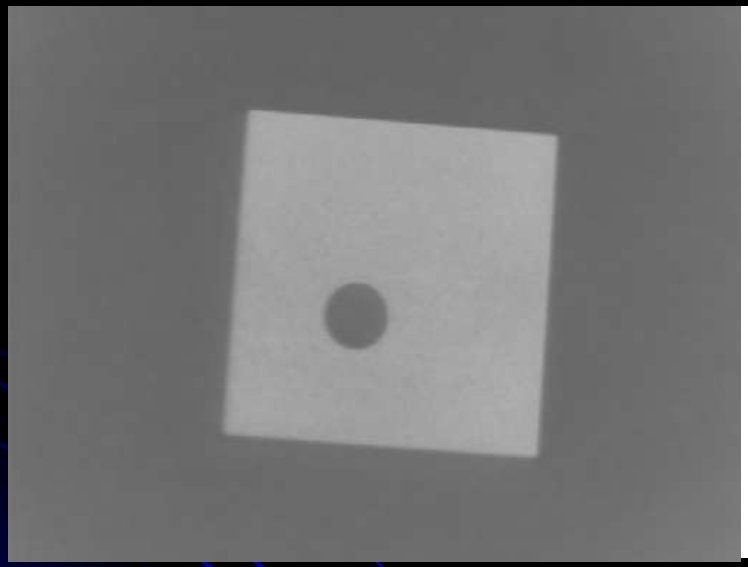


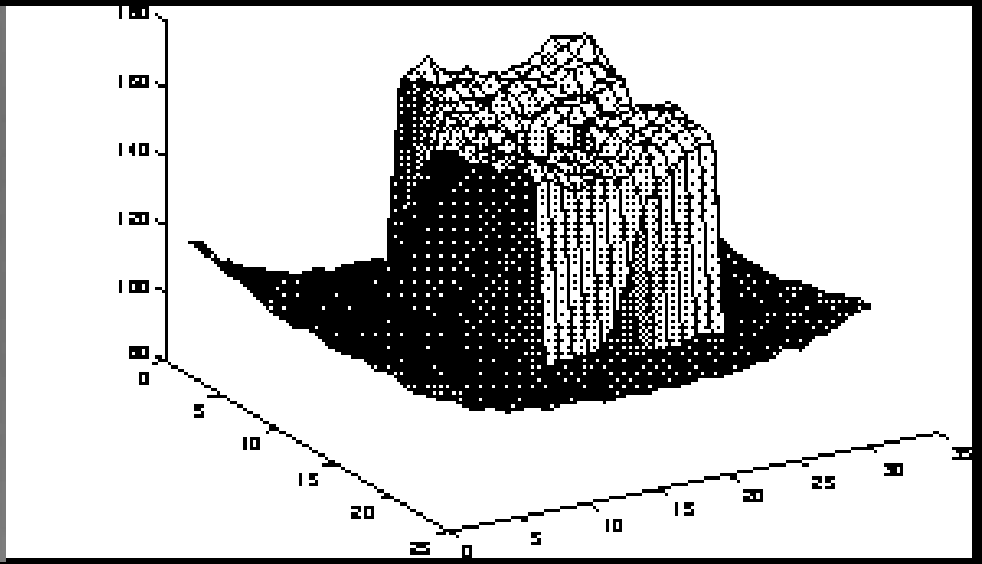
obraz



3D reprezentácia

Obraz

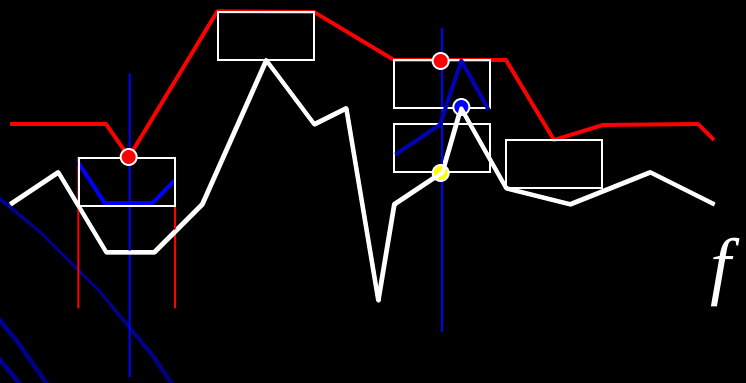


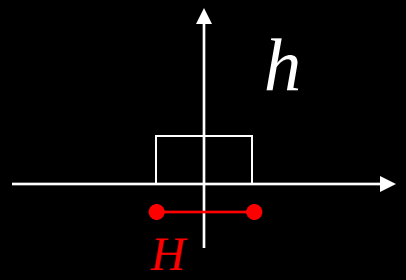


Dilatácia

$$(f \oplus h)(x,y) = \max_{(r,s)\in H} f(x-r, y-s) + h(r,s)$$

$$(f \oplus h)(x) = \max_{r\in H} f(x-r) + h(r)$$

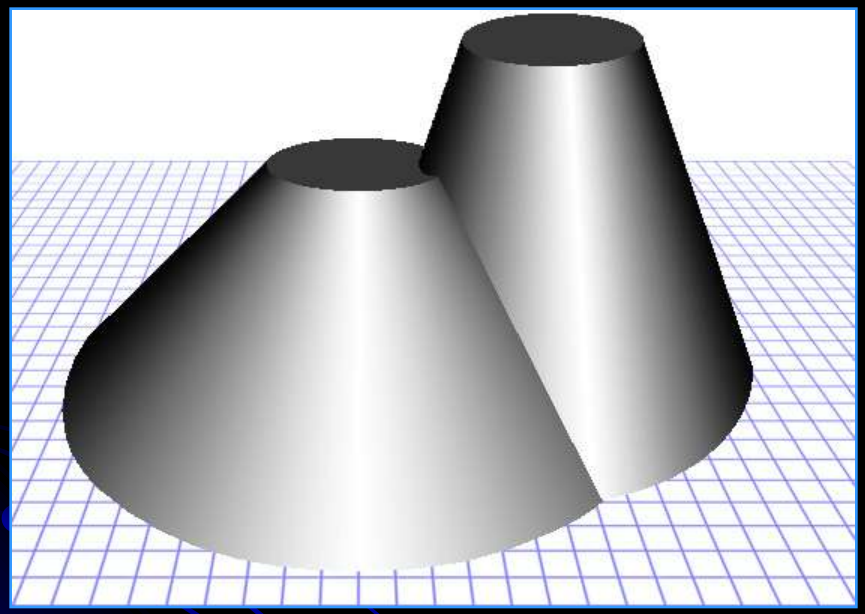




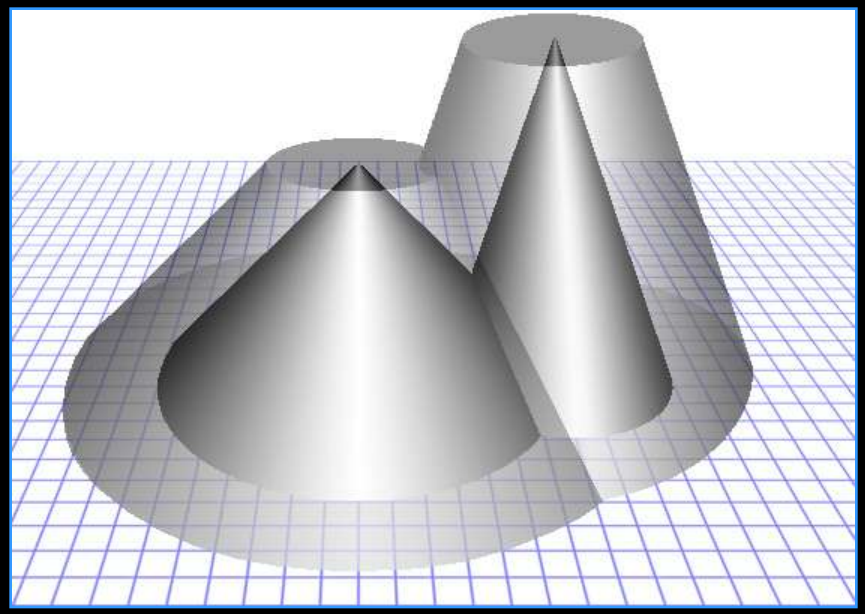
H(x) - "structuring functions"

Dilatácia

ilustrácia v 3D



Dilatácia



Dilatácia vs. originál

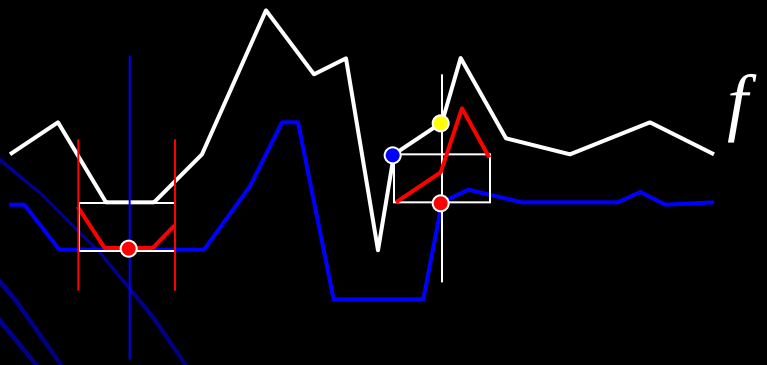
Dilatácia

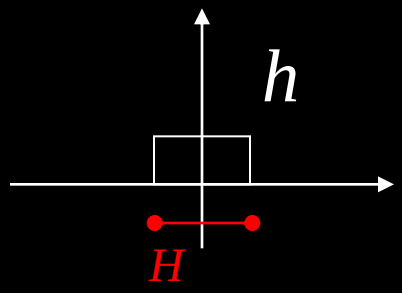
Zjasňuje obrázok – zvyšuje intenzitu

Erózia

$$(f \ominus h)(x,y) = \min_{(r,s)\in H} f(x+r, y+s) - h(r,s)$$

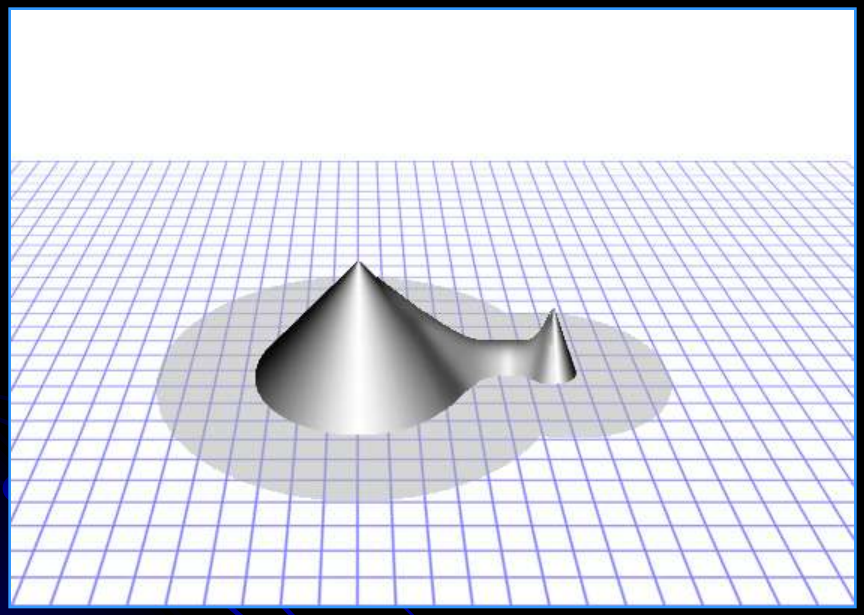
$$(f \ominus h)(x) = \min_{r\in H} f(x+r) - h(r)$$



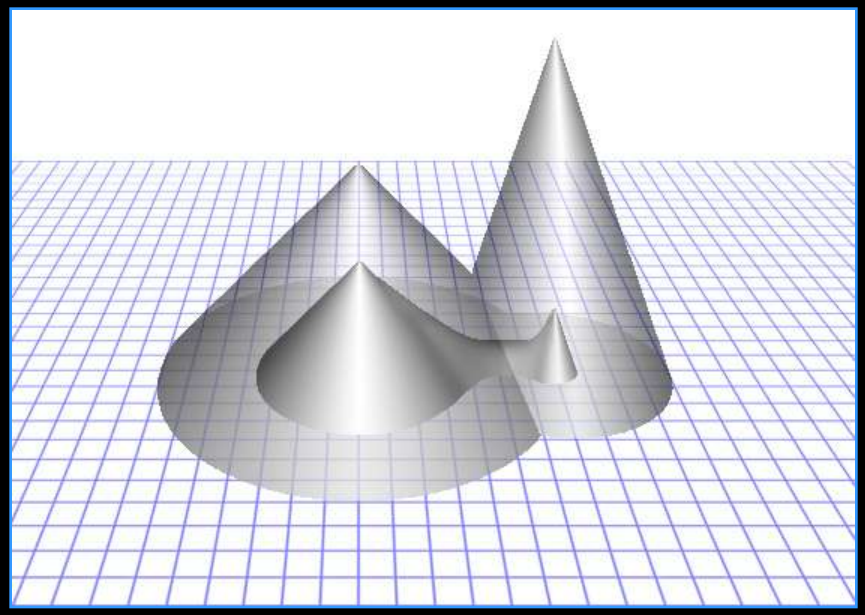


Erózia

ilustrácia v 3D



Erózia



Erózia vs. originál

Erózia

Ztmavuje obrázok – znižuje intenzitu

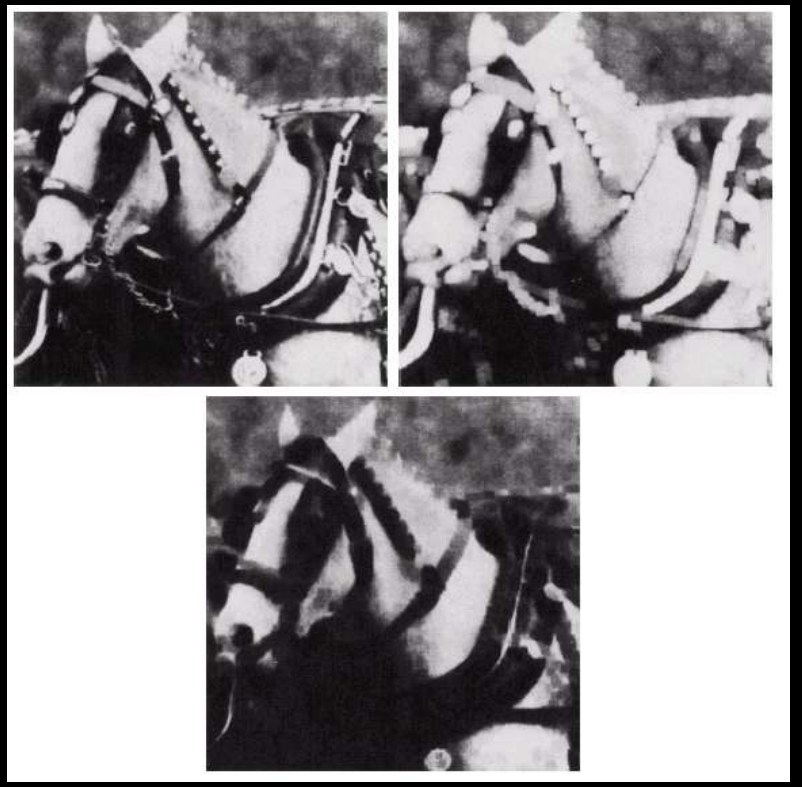
DE zhrnutie

D:

- Jasnejší obrázok
- Redukuje (odstraňuje) tmavé detaily

E:

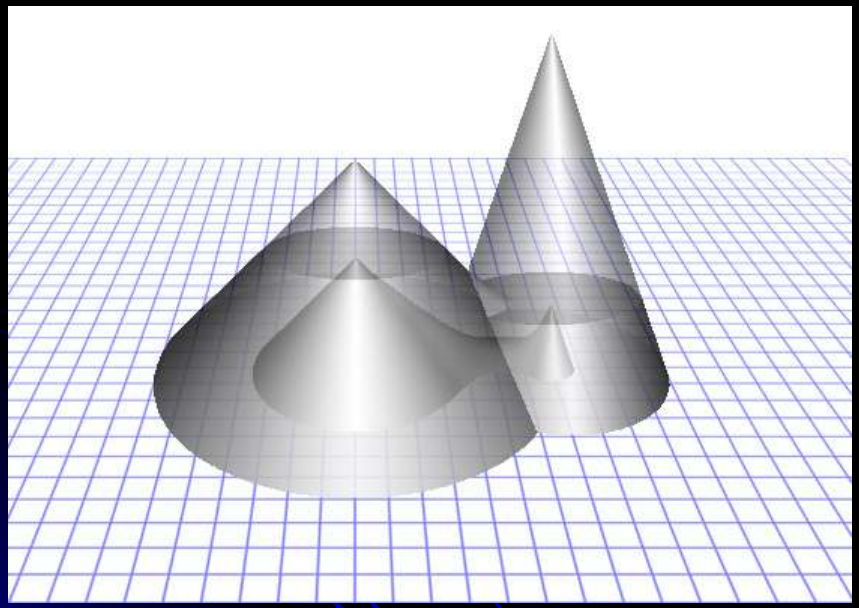
- Tmavší obrázok
- Redukuje (odstraňuje) svetlé detaily

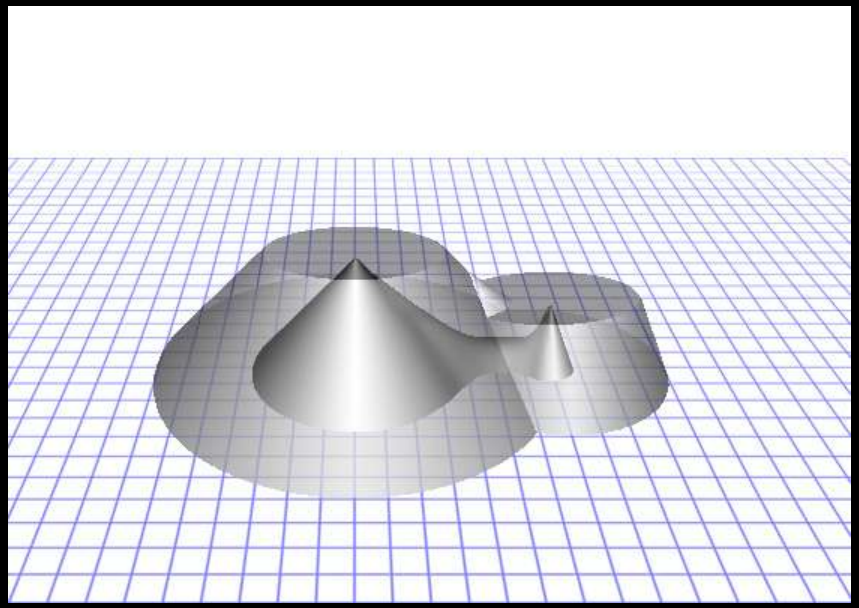


Otvorenie

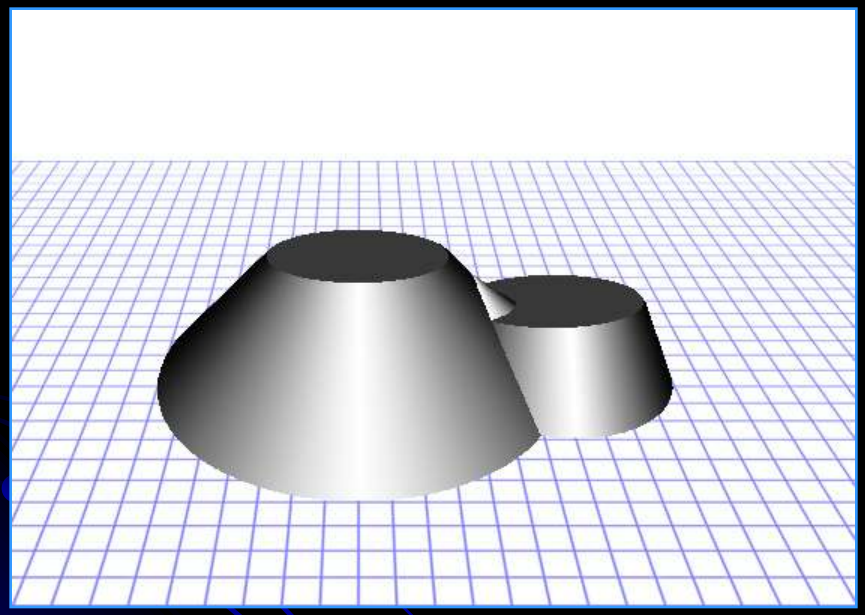
$$A \circ B = (A \ominus B) \oplus B$$

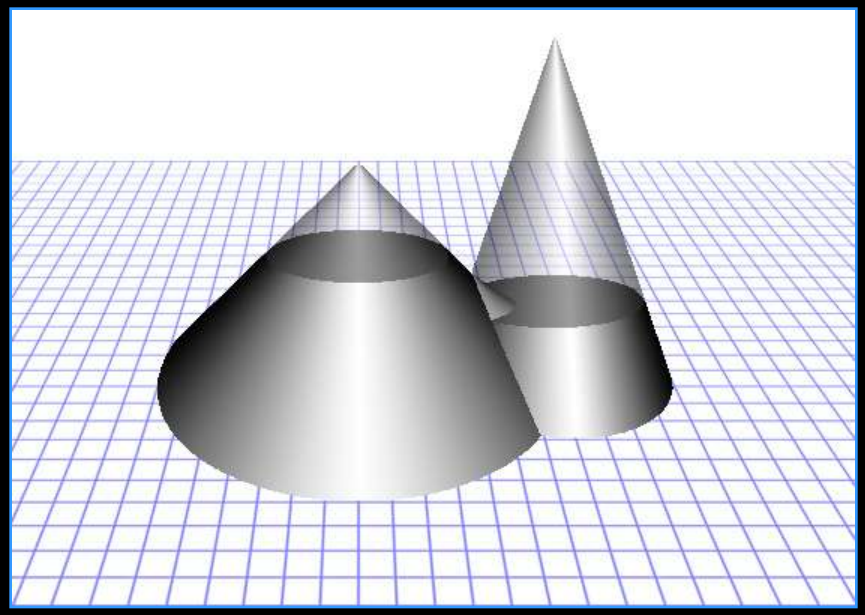
Otvorenie





Otvorenie



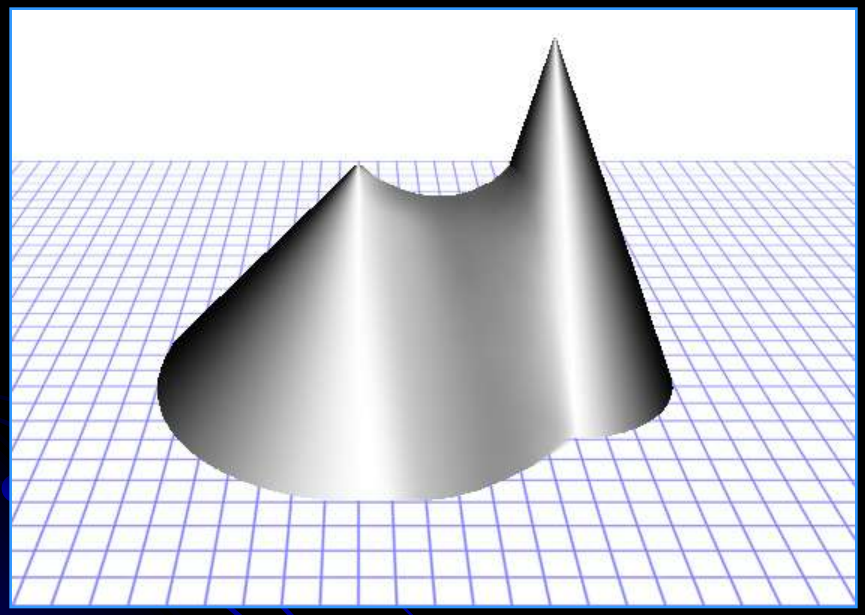


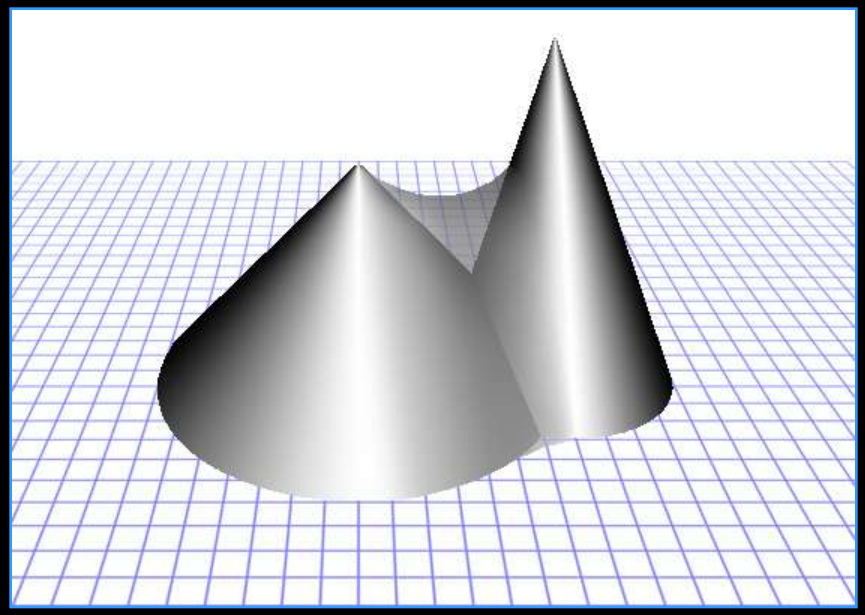
Otvorenie

Uzavretie

$$A \bullet B = A \oplus B \ominus B$$

Uzavretie





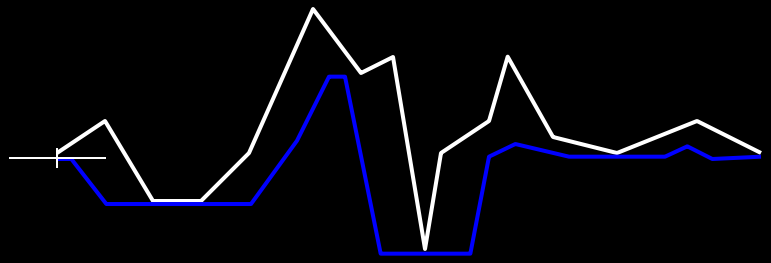
Uzavretie

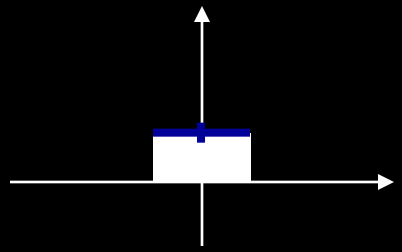
$$A \circ B = (A \ominus B) \oplus B$$

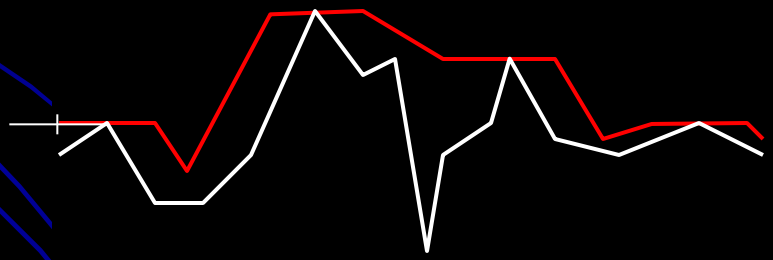
OU

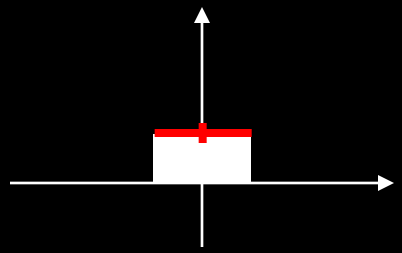
$$A \bullet B = (A \oplus B) \ominus B$$

otvorenie



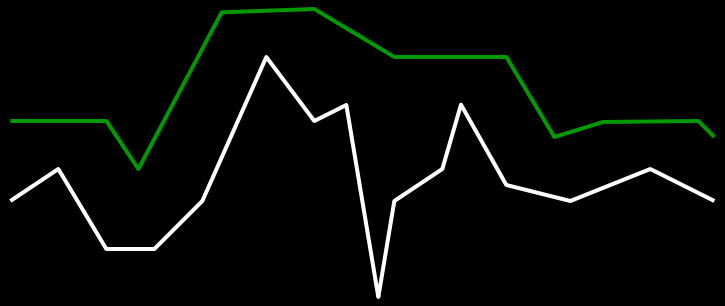




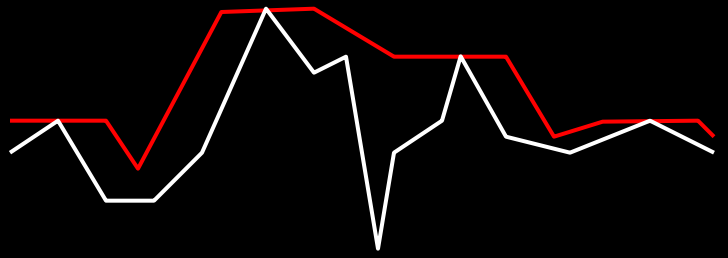


uzavretie

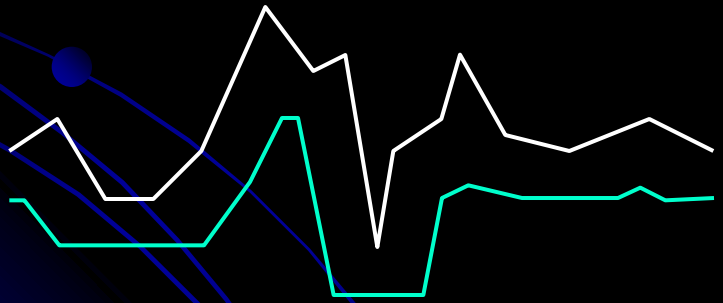
Zhrnutie



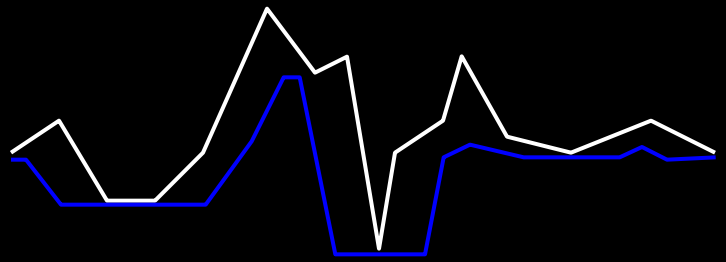
D



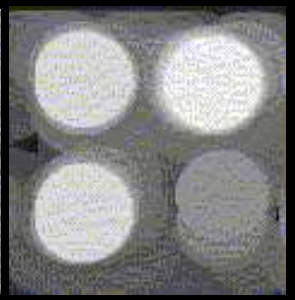
U



E



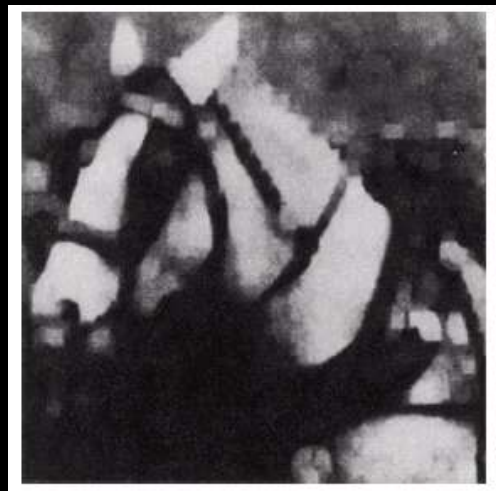
O

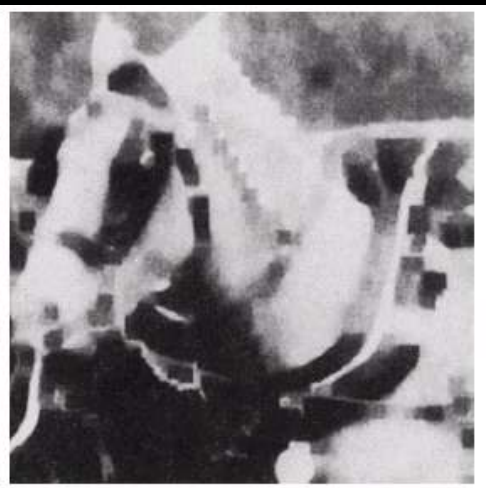


OU zhrnutie

O: odstraňuje malé svetlé objekty

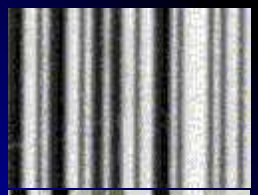
odstraňuje šum



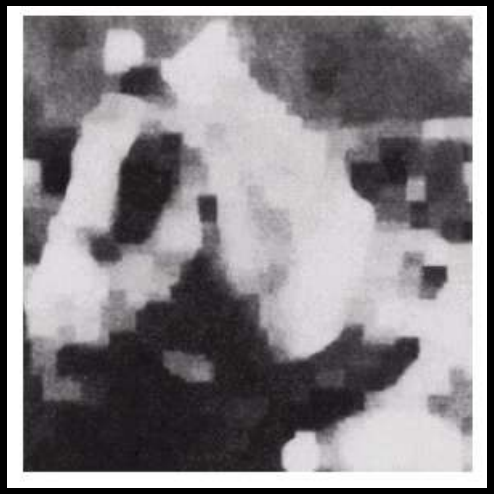


U: spája svetlé objekty

redukuje malé tmavé oblasti



Príklad použitia



$(A \circ B) \bullet B$ – filtrácia obrazu

Opakovanie

⊕ dilatácia

$$A \oplus B = \bigcup_{b \in B} A_b$$

⊖ erózia

$$A \ominus B = \bigcap_{b \in B} A_{-b}$$

∘ otvorenie

$$A \circ B = (A \ominus B) \oplus B$$

• uzavretie

$$A \bullet B = (A \oplus B) \ominus B$$

⊗ hit-and-miss

$$A \otimes B = (A \ominus B_1) \cap (A^C \ominus B_2)$$

Farebná morfológia

Šedotónový obraz

$X = \{ f(\mathbf{a};X(\mathbf{a})) \mid \mathbf{a} \in E_{n-1},\ X(\mathbf{a}) \in \Re \cup \{\infty\} \cup \{-\infty\} \}$

Dilatácia $(f \oplus h)(x,y) = \max_{(r,s)\in H} f(x-r, y-s) + h(r,s)$

Erózia $(f \ominus h)(x,y) = \min_{(r,s)\in H} f(x+r, y+s) - h(r,s)$

Farebný obraz

$X = \{ f(\mathbf{a};X(\mathbf{a})) \mid \mathbf{a} \in E_{n-1},\ X(\mathbf{a}) \in \Re^3 \cup \{\infty\} \cup \{-\infty\} \}$

min/max ???

Multivariate ordering

- Ako nájst inf/min a sup/max vektorov ?

$x_1=(1,1)$
$x_2=(5,3)$
$x_3=(9,2)$
$x_4=(3,3)$
$x_5=(5,4)$
$x_6=(6,5)$
$x_7=(6,8)$

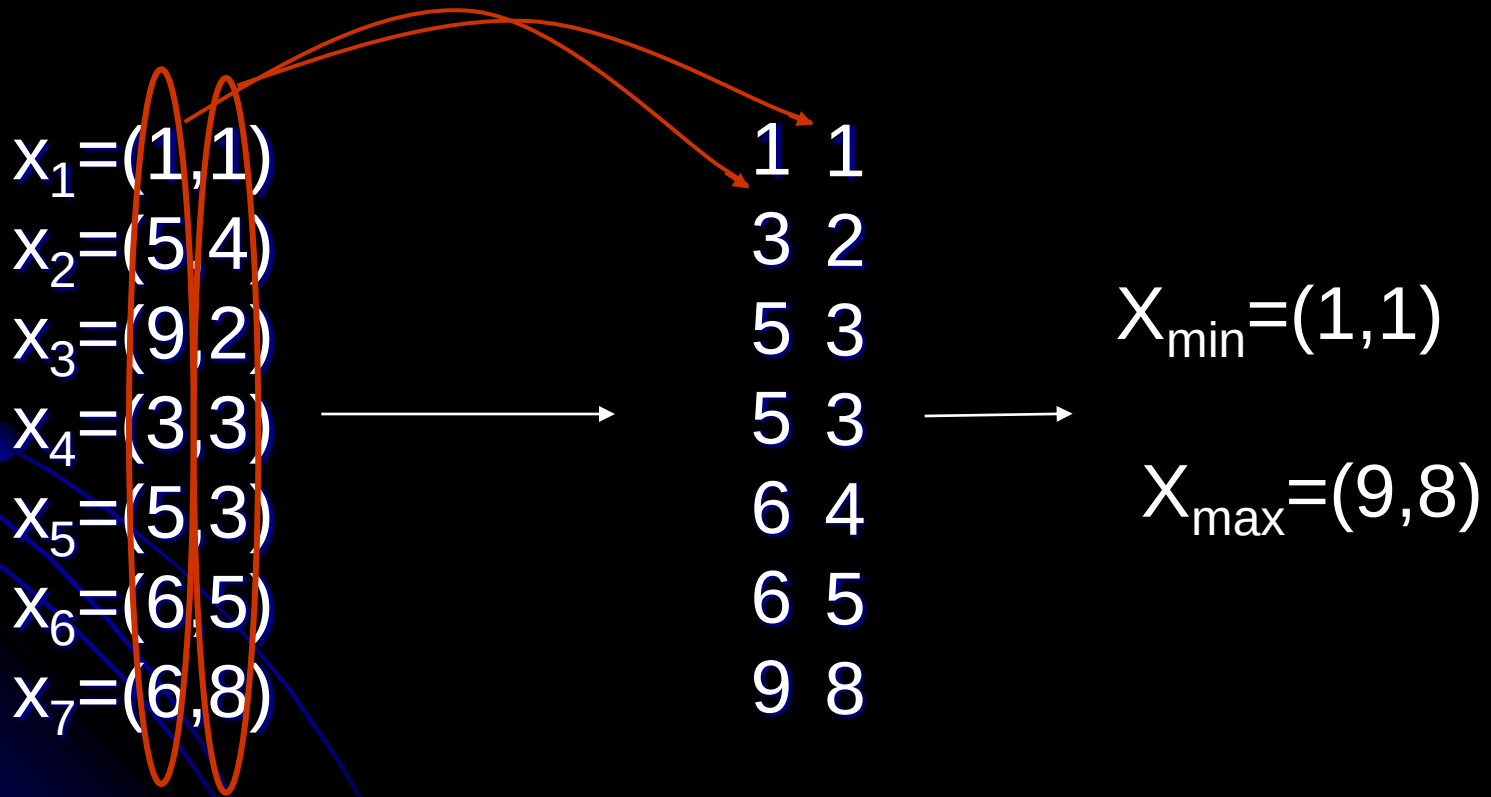
Min/max ⟶ ?

Multivariate ordering

- Marginal ordering – M-ordering
- Conditional ordering – C-ordering
- Partial ordering – P-ordering
- Reduced ordering – R-ordering

Multivariate ordering
Marginal ordering – M-ordering

- Usporiadam jednotlivé zložky



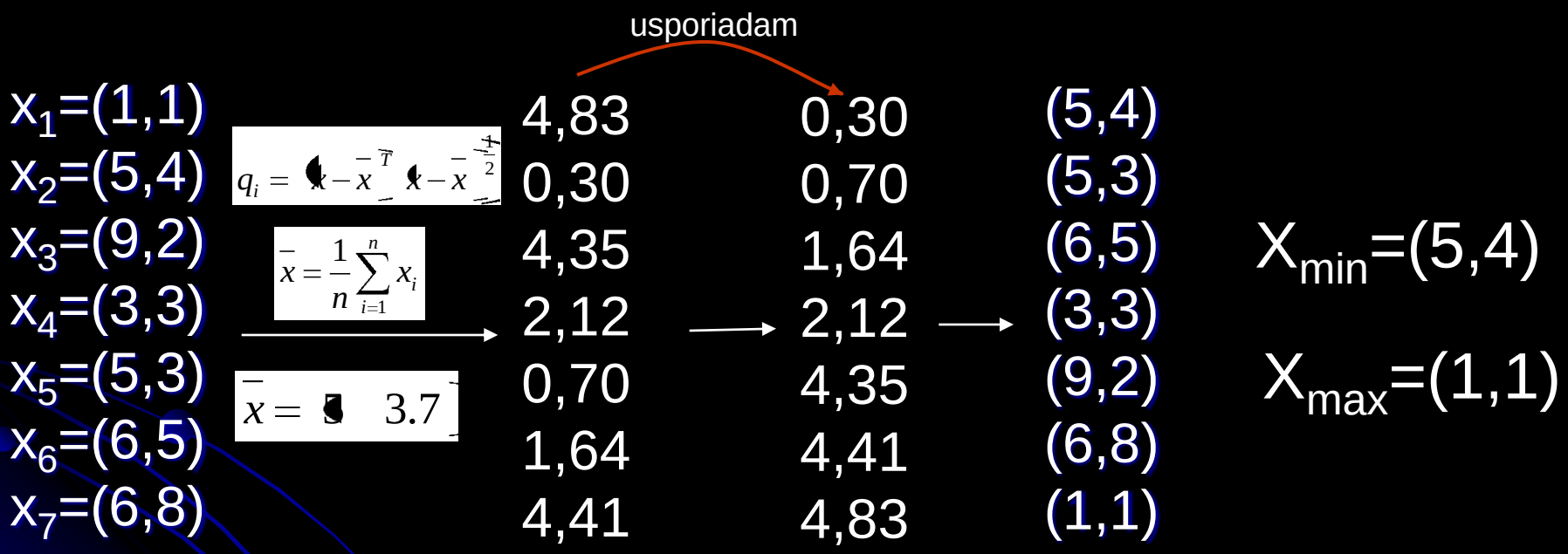
Môžu vzniknúť nové vektory!

Multivariate ordering
Partial ordering – P-ordering

Dáta sa rozdelia do podmnožín pomocou postupného vytvárania minimálneho konvexného obalu dát

Multivariate ordering
Reduced ordering – R-ordering

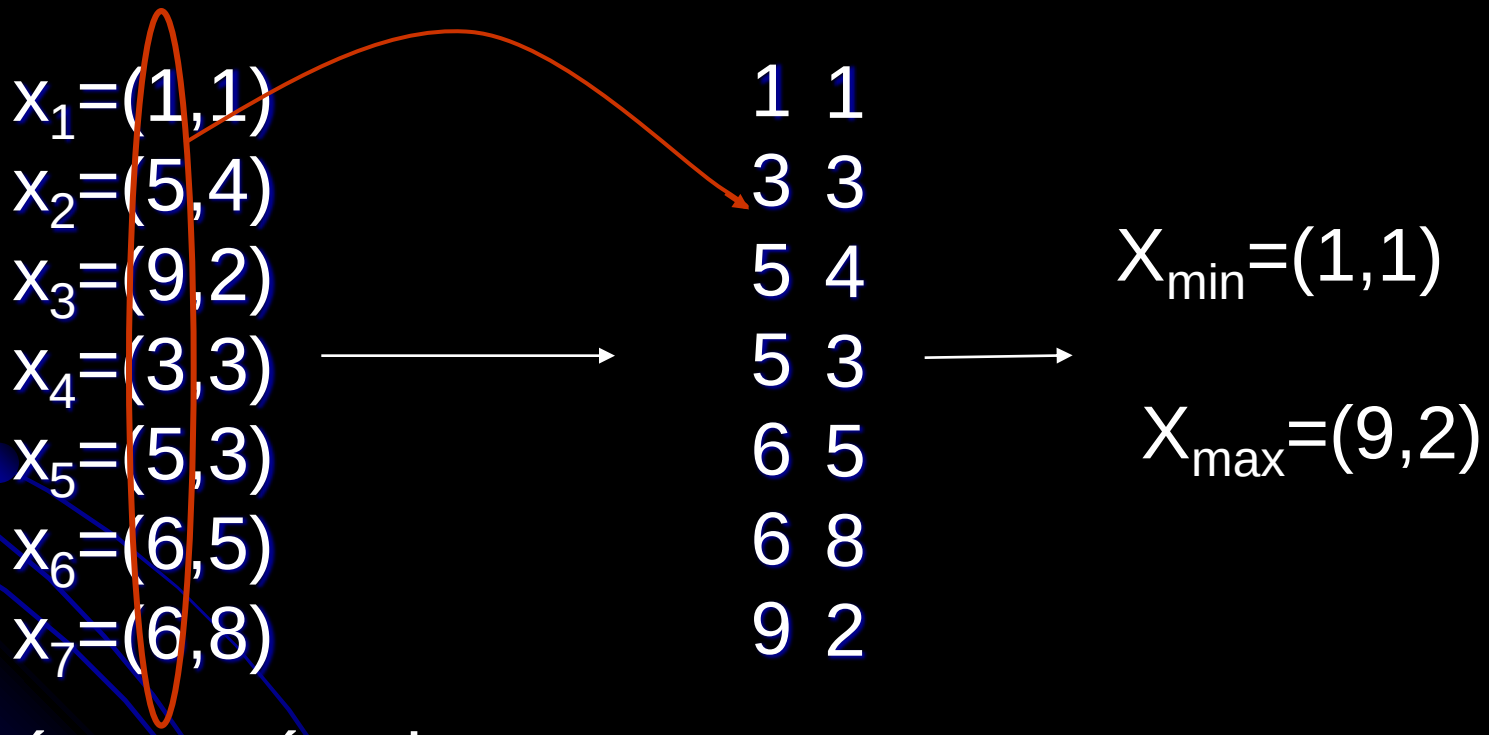
- Zredukujem vektor do skalára redukčnou funkciou



Vhodné na identifikáciu odľahlých bodov - oulier.

Multivariate ordering
Conditional ordering – C-ordering

- Usporiadam podľa prvej zložky



Nevytvára nové vektory.

Predpokladá že prvá zložka nesie najviac informácie.

Conditional ordering – C-ordering

lexikografické usporiadanie

$$(a_1, b_1, c_1) < (a_2, b_2, c_2) \quad \text{if} \quad \begin{cases} a_1 < a_2 \\ \text{or} \\ a_1 = a_2 \quad \text{and} \quad b_1 < b_2 \\ \text{or} \\ a_1 = a_2 \quad \text{and} \quad b_1 = b_2 \quad \text{and} \quad c_1 < c_2 \end{cases}$$

- Kladie dôraz na poradie zložiek
- Predpokladá že najviac informácie obsahuje prvá zložka podľa ktorej triedime
- Vylepšenie použitím napríklad PCA

Variácie lexikografického usporiadania

α – lexikografické usporiadanie

$$\forall \boldsymbol{v}, \quad \boldsymbol{v}' \in \mathbb{R}^n, \boldsymbol{v} < \boldsymbol{v}'$$
$$\Longleftrightarrow \begin{cases} v_1 + \alpha < v_1', & \text{or} \\ v_1 + \alpha \geqslant v_1' & \text{and} \quad [v_2, \ldots, v_n]^{\mathrm{T}} <_L [v_2', \ldots, v_n']^{\mathrm{T}} \end{cases}$$

kde $\alpha \in \Re^+$

Presúva porovnanie častejšie na druhú zložku

Variácie lexikografického usporiadania

- α-trimmed lexicographical extrema

  Hľadám max z *k* vektorov:

  Usporiadam podľa 1. zložky a nechám si iba α x k vektorov kde $\alpha \in \langle 0,1 \rangle$ novú množinu vektorov usporiadam podľa 2. zložky

- α-modulus lexicographical ordering

$$\forall \boldsymbol{v},\ \boldsymbol{v}' \in \mathbb{Z}^n,$$

$$\boldsymbol{v} < \boldsymbol{v}' \iff [\lceil v_1/\alpha \rceil, v_2, \ldots, v_n]^{\mathrm{T}} <_L [\lceil v'_1/\alpha \rceil, v'_2, \ldots, v'_n]^{\mathrm{T}}$$

Farebná morfológia

Sekvenčný prístup:

Jednotlivé kanály samostatne

Intenzita R,G,B – min/max

Lab, HSV – L, S, V – OK

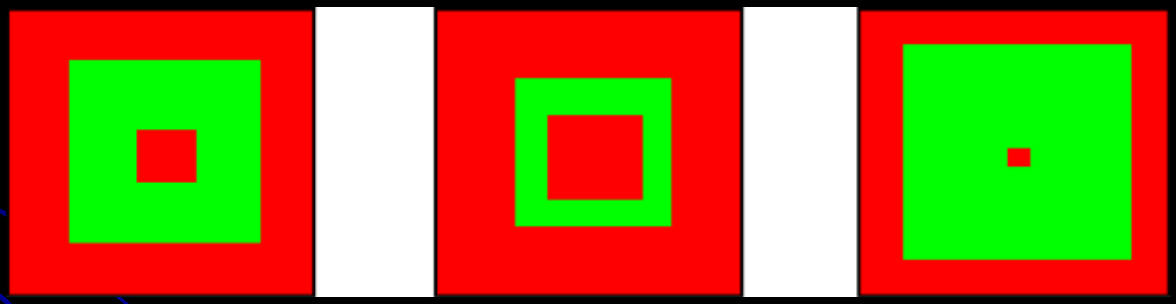
H, a, b – červená > zelená ???

Problém? (R $\oplus$ C, G $\oplus$ C, B $\oplus$ C)

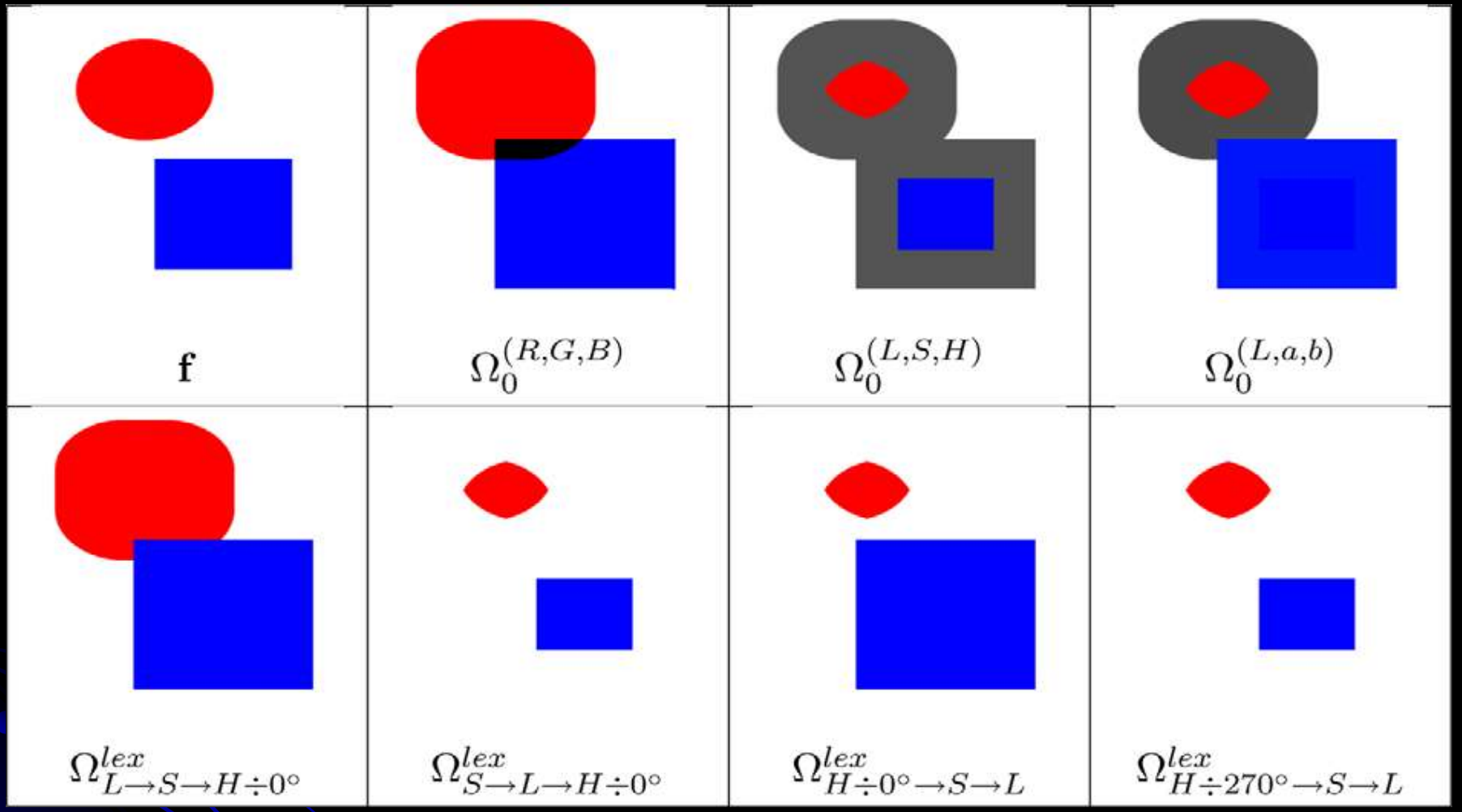
Farebná morfológia

$$\varepsilon_H f(\mathbf{x}) = \{\mathbf{c}_j : D(H_j, H_0) = \sup[D(H_i, H_0)], \mathbf{c}_i \in B_\mathbf{x}\}$$

$$\delta_H f(\mathbf{x}) = \{\mathbf{c}_j : D(H_j, H_0) = \inf[D(H_i, H_0)], \mathbf{c}_i \in B_\mathbf{x}\}$$



Original image (left), results of applying a vectorial dilation based on a lexicographical ordering (R->G->B, middle) (G->R->B, right) with a 21 x 21 square SE.



Comparison of colour erosion for the image f (the structuring element B is a square of size n = 35), $\varepsilon_{\Omega,nB}(f)$, using different orderings Ω: 3 examples of marginal orderings Ω_0 in the RGB, LSH and L*a*b* colour spaces and 4 examples of total lexicographic-based orderings Ω^{lex} in LSH giving the priority to the L, or to S, or to h_0-centred H (origins in the red 0 and in the purple 270).

The erosion and dilation results of a part of the macaws image with a square shaped SE of size 9 x 9 pixels, using a-modulus lexicographical ordering (in HSL color space)