PROCEEDINGS OF SYMPOSIUM ON COMPUTER GEOMETRY SCG ' 2007, VOLUME 16,
pp. xy -xz

BÉZIER CURVES INTERSECTION USING MONGE PROJECTION

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Abstract. Intersection of spatial curves became general problem in the sphere of geometric modeling and computational geometry. To find a set of points of the curve/curve intersection could be performed by several algorithmic techniques. At the present time there exist several different approaches to this problem but the endeavor is to avoid difficulties in calculation which are mainly results of polynomial representation higher degree curves. We stay indeed at the original polynomial approach, e.g. Bézier clipping method, established by Nishita et al. in [1990]. Presented paper describes the mathematical background of this method that solves existing problem of two curves intersection on class plane Bézier curves only.

Keywords: Bézier curve, orthogonal projections, Bézier clipping, Monge projection

1 INTRODUCTION

Raytracing has become a popular method for generating and rendering high quality images and movies. The basic requirement for ray tracing is the computation of intersection between rays and object. As a simple and powerful technique resolving the problem of intersection was proposed Bézier clipping method, originally presented by Nishita et al. 1990.

In this paper we discuss display method by using Bézier clipping, see some of many references [Efr05], [Kud06], [Nis98], [Pal06], …

1.1 Outline

The content of this paper is organized as follows. Section 2 presents basic ideas behind the most commonly used method in technical-geometrical practice, i.e. Mongean Method. Section 3 deals some of basic theorems of Monge projection with and a corollary is derived. Bézier clipping method originally presented by Nishita et al. [1990] is described in Section 4. Section 5 opens problem of space nonrational Bézier curves intersection in a touch with Monge projection. Conclusions and future work are presented in Section 6.

2 MONGE PROJECTION

In the field of descriptive geometry became Mongean method the most commonly used projection in technical-geometrical practice [Cen59], [Lor33], [Pal04]. We remind that the Monge projection is a double orthogonal projection upon a pair of perpendicular planes which intersect in a line (Fig. 1).

Theorem. *To a general point **P** of space $\mathbb{E}^3$ there belongs, in the drawing plane, a pair of points $\boldsymbol{P}_1$, $\boldsymbol{P}_2$, which two points lie on the same perpendicular to the ground line; and, conversely, to a pair of points so related there is but one point in space $\mathbb{E}^3$, namely that to which this pair belongs.*

Definition. *The Mongean Method of Representation possesses the property of unambiguous correspondence.*

Let $\varphi_1 : \mathbb{E}_3 \to \mathbb{E}_2 = \pi$ $[\![\varphi_2 : \mathbb{E}_3 \to \mathbb{E}_2 = \upsilon]\!]$ be the orthogonal projection upon a plane of projection π (first plane of projection) $[\![\upsilon$ (second plane of projection)$]\!]$. The point $\varphi_1(\mathbf{X}) = \mathbf{X}_1 = [x, y]$ $[\![\varphi_2(\mathbf{X}) = \mathbf{X}_2 = [x, z]]\!]$ is the orthogonal projection of the point $\mathbf{X} = [x, y, z] \in \mathbb{E}^3$ upon the plane of projection $\pi[\![\upsilon]\!]$.

Let $\varphi = \varphi_1 \times \varphi_2 : \mathbb{E}^3 \to \mathbb{E}^2 \times \mathbb{E}^2 = \pi \times \upsilon = \varepsilon$ be the Monge projection and points $\mathbf{X}_1$, $\mathbf{X}_2$ are the orthogonal projections upon plane of projection π and υ respectively. After the forming a union of planes of projection π^0 (plane π is turned around the axis $x_{1,2}$ about 90° to the position when half-plane π^+ coincide with half-plane υ^- and half-plane π^- coincide with half-plane υ^+, see Fig.1) and υ into the drawing plane ε, the coordinates of $\varphi(\mathbf{X}) \in \varepsilon$ will be as follows:

$$\varphi : \mathbf{X} \to \varphi(\mathbf{X}) = \left[\varphi_1^0(\mathbf{X}), \varphi_2(\mathbf{X})\right] = \left[(\psi \circ \varphi)(\mathbf{X}) \times \varphi_2(\mathbf{X})\right] = [x, y, z] \in \varepsilon, \quad (1)$$

where $\psi : \pi \to \upsilon$ is the above rotation and the coordinates of $\varphi(\mathbf{X})$ are using as in Fig. 1. For more details see e.g. [Lor33], [Kra91], [Pal04].

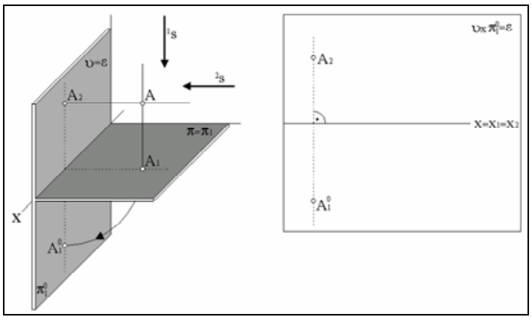


Figure 1. Union of planes of projection to form a drawing plane.

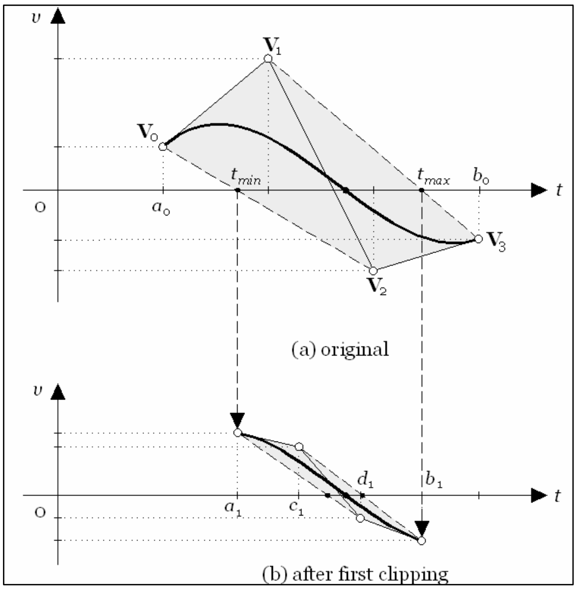


Figure 2. PT Algorithm.
(a) ***Original.*** **(b)** ***After first clipping*.**

3 BÉZIER CURVES AND MONGE PROJECTION

The Monge projection is a projection of the Euclidean space $\mathbb{E}^3$ *upon* the drawing plane ε. A Bézier curve $\mathbf{P}(u)$ of degree n is defined of the parameter domain $\langle a, b\rangle$ is represented in *2D(x,y)* coordinate space by equation

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{V}_i B_i^n(u) = \left[\sum_{i=0}^{n} x_i B_i^n(u), \sum_{i=0}^{n} y_i B_i^n(u)\right], \quad (2)$$

and in *3D(x,y,z)* coordinate space

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{V}_i B_i^n(u) = \left[\sum_{i=0}^{n} x_i B_i^n(u), \sum_{i=0}^{n} y_i B_i^n(u), \sum_{i=0}^{n} z_i B_i^n(u)\right], \tag{3}$$

where $a \le u \le b$, $\mathbf{V}_i$, $i \in \{0,1,...,n\}$ are the control points, and $B_i^n(u)$ are the Bernstein polynomials, see e.g. [Far93], [Kud04], [Pal07].
From the above conditions we get a derivation for spatial Bézier curves in the Monge projection (for drawing plane ε).

Theorem. *Let* $\varphi : \mathbb{E}_3 \to \mathbb{E}_2 \times \mathbb{E}_2$ *be the Monge projection. Then* $\mathbf{P}_1(u) = \left[\sum_{i=0}^{n} x_i B_i^n(u), \sum_{i=0}^{n} y_i B_i^n(u)\right]$, $u \in \langle a,b \rangle$, $\mathbf{P}_2(u) = \left[\sum_{i=0}^{n} x_i B_i^n(u), \sum_{i=0}^{n} z_i B_i^n(u)\right]$, $u \in \langle a,b \rangle$ *are the orthogonal projections of the curve* $\mathbf{P}(u)$ *of the form* (3) *upon the planes of projections* π *and* υ *respectively and according to* (1) *we make a pre-arrangement*

$$\varphi(\mathbf{P}(u)) = \left[\sum_{i=0}^{n} x_i B_i^n(u), \sum_{i=0}^{n} y_i B_i^n(u), \sum_{i=0}^{n} z_i B_i^n(u)\right] \in \varepsilon, \; u \in \langle a,b \rangle. \tag{4}$$

4 ORIGINAL BÉZIER CLIPPING METHOD

The original Bézier clipping method was presented by Nishita et al. [1990]. This part explains the ideas of this method, e.g. how to determine and compute the intersection of two planar Bézier curves in the rendering process [Nis98]. To process this, first we have to realize some pre-computations. We present two algorithms, ***Polynomial – t parametric Axis Algorithm*** *(PT Algorithm)* and ***Curve – Line Algorithm*** *(CL Algorithm)* respectively. Bézier clipping in the context of plane curves intersection in this paper is an interactive method which takes advantages of the Convex Hull of Bézier curves and the Variation Diminishing Property. The detailed explanation is necessary to better understanding the process in $\mathbb{E}^3$ for finding curve/curve intersection using Monge projection [Cen59], [Lor33].

4.1 Preliminary consideration

Convex Hull Property (CHP): for all $t \in \langle 0,1 \rangle$, $\mathbf{P}(u) \in \{\mathbf{V}_0, \mathbf{V}_1, \dots, \mathbf{V}_n\}$. It means that every point of a Bézier curve is enclosed by the convex hull of its defining control points.
Variation Diminishing Property (VDP): for a planar Bézier curve $\mathbf{P}(u)$, the VDP states that the number of intersections of a given line with $\mathbf{P}(u)$ is less than or equal to the number of intersections of the line with control polygon $\mathbf{V}_0\mathbf{V}_1 \dots \mathbf{V}_n$.
Distance Function. Suppose the line ℓ in the plane $\mathbb{E}^2$ has the equation:

$$\ell: ax + by + c = 0, \qquad a^2 + b^2 = 1 \tag{5}$$

with a unit normal vector. The signed distance from any point (x, y) to the line ℓ is expressed by

$$d(x,y) = \frac{ax + by + c}{a^2 + b^2} = ax + by + c \tag{6}$$

By substituting the point on the Bézier curve (2) into (6) we get

$$d(u) = a\sum_{i=0}^{n} B_i^n(u) v_i^x + b\sum_{i=0}^{n} B_i^n(u) v_i^y + c\sum_{i=0}^{n} B_i^n(u) = \sum_{i=0}^{n} B_i^n(u) d_i, \tag{7}$$

where $d_i = av_i^x + bv_i^y + c$. The function $d(u)$ is called the ***distance function*** and the scalar values d_i represent the ***signed distances*** from the control points $\mathbf{V}_i = \left(v_i^x, v_i^y\right)$ to the line ℓ. The function $d(u)$ is a ***Bézier polynomial function***.

Fat Line. Let $\mathbf{V}_0, \mathbf{V}_1, \dots, \mathbf{V}_n$ be the Bézier control points in the plane. The ***fat line*** of the Bézier curve is each strip in the plane with boundary lines to be parallel to the line $\ell = \mathbf{V}_0\mathbf{V}_n$. To restrict the number of fat lines of $\mathbf{P}(u)$ we suggest the fat line L of the cubic Bézier curve of the form (2), as the set of points $\mathbf{X} \in \mathbb{E}^2$ which satisfy $d_{\min} \le d_x \le d_{\max}$, where $d_{\min} = \min\{d_0, d_1, \dots, d_n\}$, $d_{\max} = \max\{ d_0, d_1, \dots, d_n\}$ and d_x, d_i, $i \in 0,1, \dots, n$, are the signed distances (7) from the points $\mathbf{X}, \mathbf{V}_0, \mathbf{V}_1, \dots, \mathbf{V}_n$ to the line $\ell = \mathbf{V}_0\mathbf{V}_n$. The line ℓ is represented by (5), so the boundary lines ℓ', ℓ'' of the fat line L are expressed as:

$$\ell' : ax + by + c + d_{\max} = 0,\ \ell'' : ax + by + c + d_{\min} = 0. \tag{8}$$

All the points of the convex hull belonging to the Bézier curve coincide with the fat line $L = |\ \ell'\ \ell''|$.

Iteration and Tolerance. The algorithms of iterations are based on an iterative process. The iteration terminates when

- the ***convex hull*** of the curve on the corresponding iteration *does not intersect the axis* – this indicates that there is **no intersection** of Bézier curve with the parametric axis or the given line.
- the ***interval of interest*** $[\boldsymbol{u}_{\min}, \boldsymbol{u}_{\max}]$ is smaller than a given ***threshold value of tolerance*** $\boldsymbol{\varepsilon}$ (Test B). The algorithm gives as a "return value" interval $[t_{\min}, t_{\max}]$ after each clipping iteration and the situation, when $[t_{\min}, t_{\max}] < \varepsilon$ we regard as a final iteration. Then the intersection is assumed to exist in the centre of the "final" interval of interest $[t^m{}_{\min}, t^m{}_{\max}]$. Note that we take the centre of interval for final, but approximate value of the root.[1]

4.2 Algorithms

Bézier clipping method consists of several steps that are explained below.

PT Algorithm (Polynomial – *t* parametric Axis Algorithm)

To find the root of the polynomial function.

<u>Input data:</u> ■ suppose we have a polynomial defined by a functional equation

$$y = v(t) = \mathrm{a}_0 + \mathrm{a}_1 t + \ \dots\ + \mathrm{a}_n t^n,\ t \in \langle a, b\rangle,$$

■ parametric axis t by equation

$$y = 0 \quad (ax + by + c = 0, \text{ where } a = c = 0).$$

Step 1. Conversion of given polynomial into the parametric Bézier curve of degree n.

Step 2. We determine the convex hull of the curve segment control polygon.

Step 3. Test A - to determine whether a Bézier curve's convex hull intersects the t axis. In a positive case this gives us an interval of interest $[t_{\min}, t_{\max}]$. See Fig. 2a.

Step 4. For the values $t_{\min}$, $t_{\max}$ we have to determine the ordinates of the points on the Bézier curve $\mathbf{B}(t)$. This gives us the interval of interest on the curve which corresponds to the interval $[t_{\min}, t_{\max}]$. We conclude that the curve segments corresponding to the intervals

[1] Note: If the interval of interest does not change more than 80% the Bézier curve is subdivided at the midpoint and the algorithm is applied to each segment.

$t < t_{\min}$ and $t > t_{\max}$ don't intersect the axis t and we clip them away (with respect to VDP). See Fig. 2b.

Step 5. Test B - to compare the change of new found interval [$t^i_{\min}$, $t^i_{\max}$], i = 1, 2, …, m to the original interval [a, b], or to the previous interval [$t^i_{\min}$, $t^i_{\max}$] because of [$t^m_{\min}$, $t^m_{\max}$] $\subseteq \ldots \subseteq$ [$t^i_{\min}$, $t^i_{\max}$] $\subseteq$ [$t_{\min}$, $t_{\max}$] $\subseteq$ [a, b]. Figure 2 shows the first clipping iteration. The Bézier clipping terminates when [Chap. 4.1]:

a) the convex hull of Bézier curve on the corresponding iteration does not intersect t axis.

b) the length of the interval [$t_{\min}$, $t_{\max}$] is smaller then a threshold value ε.

<u>Output data:</u> The root(s) of the polynomial function.

CL Algorithm (Curve – Line Algorithm)

To find an intersection of planar Bézier curve and a line.

<u>Input data:</u>
- ■ Bézier curve $\mathbf{B}(t) = \sum_{i=0}^{n} B_{i,n}(t)\mathbf{V}_i$, $t \in \langle a,b \rangle$,
- ■ line ℓ by its implicit equation ℓ: $ax + by + c = 0$, and the coefficients a, b, c are modified to the form: $a^2 + b^2 = 1$.

Step 1. The signed distance $d_i \in \mathrm{R}$ from the control points $\mathbf{V}_i$ to the given line ℓ. [Chap. 4.1]

Step 2. We specify the "new" control points $\mathbf{D}_i = [i/n; d_i]$ for i = 0, 1, … , n.

Step 3. We apply *PT algorithm* for function $d(t)$.

<u>Output data:</u> Intersection points of Bézier curve and the line ℓ.

CC Algorithm (Curve – Curve Algorithm)

To find an intersection of two planar Bézier curves.

<u>Input data:</u>
- ■ Two Bézier curves $\mathbf{B}(u) = \sum_{i=0}^{n} B_i^n(u)\mathbf{V}_i$, $u \in \langle a,b \rangle$,

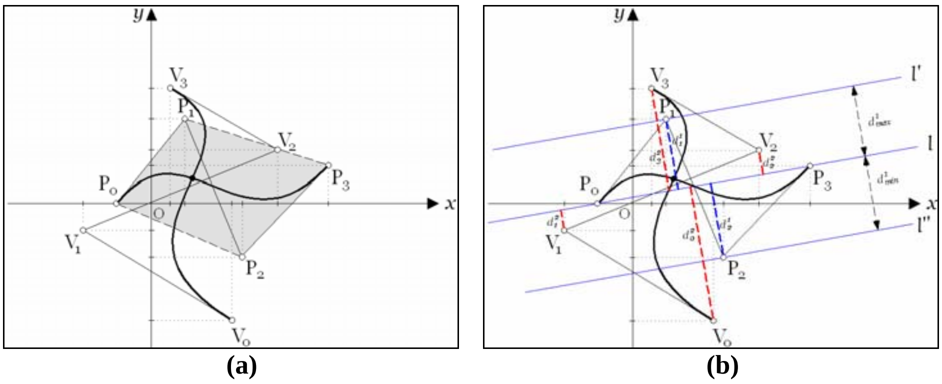
$\mathbf{C}(v) = \sum_{j=0}^{m} B_j^m(v)\mathbf{P}_j$, $v \in \langle c,d \rangle$. Fig. 3 a).

Step 1. Test C - to determine intersection of two Bézier curves convex hulls by the Min-max box method; more details in [Pal06],[Mar99].

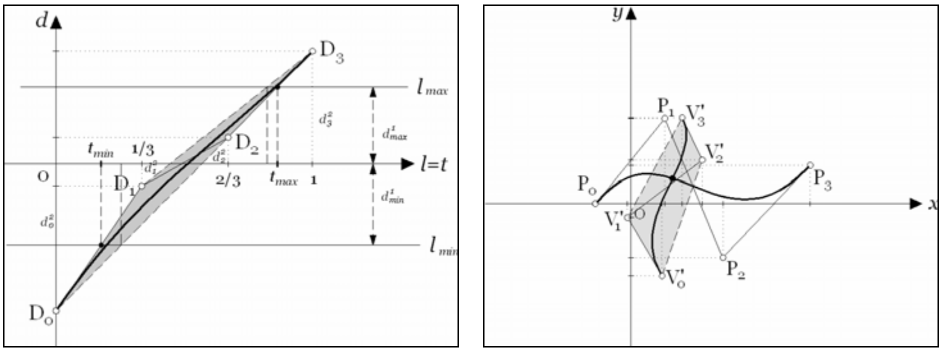
Step 2. We construct the fat line $\ell'\ell''$ [Chap. 4.1] of the curve $\mathbf{C}(v)$. See Fig. 3 b). *Remark: If the given Bézier curves are of second or third degree we can construct the Fat line to be narrower strip* [Pal06].

Step 3. Using previous algorithms or their parts we find the intersection of Bézier curve with lines ℓ', ℓ''. We practice *CL algorithm* and modified *PT algorithm*. Fig. 3 c), d).

<u>Output data:</u> Intersection points of two Bézier curves.



(a) **(b)**



(c) **(d)**

Figure 3. CC algorithm. (a) ***Given curves.*** **(b)** ***Fat Line of C(v) and Signed Distances of B(u).*** **(c)** ***New Form of Bézier curve*****. (d)** ***After First Clipping Iteration for C(v).***

5 INTERSECTION OF SPACE NONRATIONAL BÉZIER CURVES

We formulate a new technique for computing the roots of "polynomial equation systems", because of simple *3D* space decomposition ("reduction") onto two planar cases of curves intersection. Each partition of decomposition is solved separately, but the final step of given method consists of a reverse composition both parts. We come out of theorems about a pair of projections of two lines, the first in π, the second in υ, and the two so related that perpendiculars from them to the axis have a common ordinate (as in Section 4 is described) and, conversely, analogous. We get a suitable assumption to determine intersection of two curves in *3D(x,y,z)* coordinate system and conclude the corollary of these theorems (for some next similarities see [Hlu00]).

Theorem. a) *Let* ***p, q*** *be the* ***concurrent lines*** *which are not perpendicular to the ground line; their intersection point is marked* ***M****. Then their first and second projections* $\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{q_1}, \boldsymbol{q_2}$ *are all lines, no one of them is the ordinate. Hereby one and only one of the following conditions holds* [Fig.4]*:*

1. ($\boldsymbol{p_1}, \boldsymbol{q_1}$) *and* ($\boldsymbol{p_2}, \boldsymbol{q_2}$) *are pairs of concurrent lines whereby* ***points*** $\boldsymbol{M_1 = p_1 \cap q_1}$, $\boldsymbol{M_2 = p_2 \cap q_2}$ ***lie on the ordinate****,*
2. *lines* $\boldsymbol{p_1}, \boldsymbol{q_1}$ *coincide together and pair of lines* ($\boldsymbol{p_2}, \boldsymbol{q_2}$) *are concurrent lines,*
3. *pair of lines* ($\boldsymbol{p_1}, \boldsymbol{q_1}$) *are concurrent lines and lines* $\boldsymbol{p_2}, \boldsymbol{q_2}$ *coincide together.*

b) *If for the lines* $\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{q_1}, \boldsymbol{q_2}$*, each of which lie in the drawing plane and which are not ordinates, occur one of that three opportunities listed in part a), then the lines are first and second projections of concurrent lines* ***p, q****; no one of them is perpendicular to the ground line.*

Proof.[2]

[2] Proof of above theorem may be found in [Kraem91]

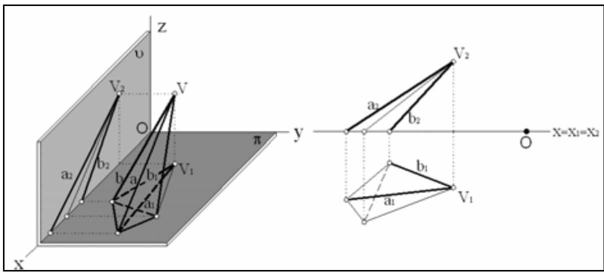


Figure 4. Projections of Concurrent Lines.

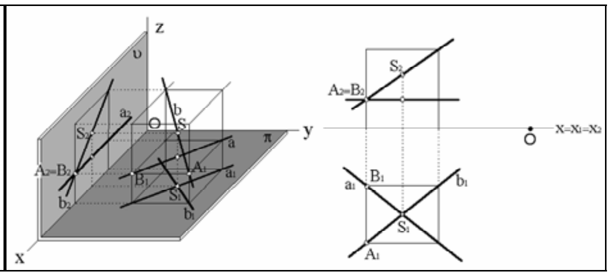


Figure 5. Projections of Skew Lines.

Theorem. a) *Let **p, q** be the **skew lines** which no one of them is perpendicular to the ground line. Then their first and second projections $\boldsymbol{p_1, p_2, q_1, q_2}$ are all lines, no one of them is the ordinate. Hereby one and only one of the following conditions holds* [Fig.5]*:*

1. $\boldsymbol{p_1, q_1}$ *are concurrent lines with point of intersection $\boldsymbol{M_1}$; $\boldsymbol{p_2, q_2}$ are concurrent lines with point of intersection $\boldsymbol{N_2}$, whereby points $\boldsymbol{M_1, N_2}$ **don't lie on the ordinate**,*
2. $\boldsymbol{p_1, q_1}$ *are various parallel lines and* $\boldsymbol{p_2, q_2}$ *are concurrent lines,*
3. $\boldsymbol{p_1, q_1}$ *are concurrent lines and* $\boldsymbol{p_2, q_2}$ *are various parallel lines.*

b) *If for the lines $\boldsymbol{p_1, p_2, q_1, q_2}$, each of which lie in the drawing plane and which are not ordinates, occur one of that three opportunities listed in part a), then the lines are first and second projections of skew lines **p, q**; no one of them is perpendicular to the ground line.*

Proof.[3]

We derivate the following corollary for curves intersection, i.e. introduce the basic idea of promising method for solving this widespread problem.

Corollary. *Let* $\mathbf{P}(u)$, $\mathbf{Q}(v)$ *be the nonrational **Bézier curves** of degree n and m respectively **in the 3D(x,y,z)** coordinate space. Then their pairs (= first and second) of projections* $\mathbf{P}_1(u)$, $\mathbf{Q}_1(v)$, $\mathbf{P}_2(u)$, $\mathbf{Q}_2(v)$ *upon a planes of projections π and υ respectively, are both the nonrational planar Bézier curves of degree n and m respectively. Hereby there can occur exact these four opportunities:*

1. *pairs of projections* $\mathbf{P}_1(u)$, $\mathbf{Q}_1(v)$ *and* $\mathbf{P}_2(u)$, $\mathbf{Q}_2(v)$ *has no intersection respectively, which makes no intersection of curves* $\mathbf{P}(u)$, $\mathbf{Q}(v)$,
2. $\mathbf{P}_1(u) \cap \mathbf{Q}_1(v) = \{\mathbf{X}_i\},\ i \in 1,\ldots$ *and* $\mathbf{P}_2(u) \cap \mathbf{Q}_2(v) = \{\ \}$, *then* $\mathbf{P}(u)$, $\mathbf{Q}(v)$ *don't intersect each other,*
3. $\mathbf{P}_2(u) \cap \mathbf{Q}_2(v) = \{\mathbf{Y}_j\},\ j \in 1,\ldots$ *and* $\mathbf{P}_1(u) \cap \mathbf{Q}_1(v) = \{\ \}$, *then* $\mathbf{P}(u)$, $\mathbf{Q}(v)$ *don't intersect each other,*
4. $\mathbf{P}_1(u) \cap \mathbf{Q}_1(v) = \{\mathbf{X}_i\},\ i \in 1,\ldots$ *and* $\mathbf{P}_2(u) \cap \mathbf{Q}_2(v) = \{\mathbf{Y}_j\},\ j \in 1,\ldots,$ *then* $\mathbf{P}(u) \cap \mathbf{Q}(v) = [\mathbf{X}_i, \mathbf{Y}_j]$ *(in respect of the parameters u, v), **intersect** each other, **if and only if** $\overline{\mathbf{X}_i\mathbf{Y}_i}$ **is the ordinate**.*

The next scheme shows a whole process of finding the intersection of the $\mathbf{P}(u)$, $\mathbf{Q}(v)$ curves:

$$\left.\begin{array}{l}
\left.\begin{array}{l}
\mathbf{P}(u)\xrightarrow{\varphi_1}\mathbf{P}_1(u)\xrightarrow{\psi}\mathbf{P}_1^0(u)\\
\mathbf{Q}(v)\xrightarrow{\varphi_1}\mathbf{Q}_1(v)\xrightarrow{\psi}\mathbf{Q}_1^0(v)
\end{array}\right\}\xrightarrow{\text{B.clipping}}\mathbf{R}_1^k(u),k\in\mathbb{N}_0\\
\left.\begin{array}{l}
\mathbf{P}(u)\xrightarrow{\varphi_2}\mathbf{P}_2(u)\\
\mathbf{Q}(v)\xrightarrow{\varphi_2}\mathbf{Q}_2(v)
\end{array}\right\}\xrightarrow{\text{B.clipping}}\mathbf{R}_2^l(u),l\in\mathbb{N}_0
\end{array}\right\}\rightarrow$$

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[3] Proof of above theorem may be found in [Kraem91]

Test [corollary], if for corresponding values of parameters u and v holds

$$\mathbf{R}_1^k(u)\mathbf{R}_2^l(u) \perp x_{1,2} \Rightarrow \mathbf{R}(u) = \mathbf{P}(u) \cap \mathbf{Q}(v) = \mathbf{R}(v)$$

φ_1 - orthogonal projection upon the plane π, $\varphi_1 : \mathbb{E}^3 \rightarrow \mathbb{E}^2 = \pi$

φ_2 - orthogonal projection upon the plane υ, $\varphi_2 : \mathbb{E}^3 \rightarrow \mathbb{E}^2 = \upsilon$

ψ - rotation of π into υ, $\psi : \pi \rightarrow \upsilon$

6 CONCLUSION AND FUTURE WORK

In this paper we describe the technique for finding intersection of spatial Bézier curves using properties of Mongean method in the context of plane curves only. There are the main themes of future work, detailed in [Pal06]:

- *Bézier clipping for non-Bézier curves*
- *Transformations of Bézier curves suffering by many disadvantages to the spline curves.*
- *Convergence of Bézier clipping*

ACKNOWLEDGEMENT

The research was supported by the VEGA Grant No. 1/3024/06.

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