DÚ2, príklad 4:

hyperbola: $Q_1: \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$

konjug. hyperbola: $Q_2: -\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0 \rightsquigarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$

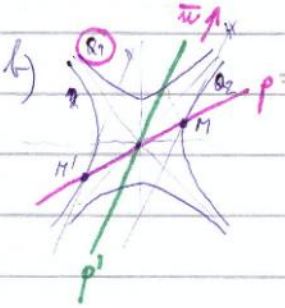
matice Q_1, Q_2:

$$Q_{1/2} = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & -\frac{1}{b^2} & 0 \\ 0 & 0 & +1/-1 \end{pmatrix}$$

a) združené priemery $\leftrightarrow$ združené smery

$$(u, v, 0) \begin{pmatrix} \frac{1}{a^2} & & \\ & -\frac{1}{b^2} & \\ & & +1/-1 \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 0 \end{pmatrix} \quad \text{alebo} \quad (u, v, 0) \begin{pmatrix} \frac{1}{a^2} & & \\ & -\frac{1}{b^2} & \\ & & +1/-1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

NEZÁVISÍ OD $+$?

b) 

$M(u, v) \rightarrow M'(-u, -v)$

dotyč. v M: $Q_2(M) = \frac{u}{a^2}x - \frac{v}{b^2}y + 1 = 0$

dotyč. v M': $Q_2(M) = -\frac{u}{a^2}x + \frac{v}{b^2}y + 1 = 0$

} rovnobežné $\bar{s} = (\frac{u}{b^2}, \frac{v}{a^2})$

priamka tvorená bodmi M, M': $\bar{s} = (-2u, -2v)$ $\bar{n} = (u, -v)$

p: $\boxed{xu - vy = 0}$

$Q_1(\frac{u}{b^2}, \frac{v}{a^2})$:

$$(\frac{u}{b^2}, \frac{v}{a^2}, 0) \begin{pmatrix} \frac{1}{a^2} & & \\ & -\frac{1}{b^2} & \\ & & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

$$\frac{u}{a^2b^2}x - \frac{v}{a^2b^2}y = 0 \quad / \cdot (a^2b^2)$$

$$\boxed{ux - vy = 0}$$

c) p' je polára smeru $\overrightarrow{MM'}$:

p': $Q_2(\overrightarrow{MM'})$: $(-u, -v, 0) \begin{pmatrix} \frac{1}{a^2} & & \\ & -\frac{1}{b^2} & \\ & & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$

$$p': \quad -\frac{u}{a^2}x + \frac{v}{b^2}y = 0$$

$p' \cap Q_2$: $x = \frac{ua^2}{vb^2}y$

$$\frac{u^2a^2}{v^2b^4}y^2 - \frac{y^2}{b^2} + 1 = 0 \quad / \cdot v^2b^4$$

$$u^2a^2y^2 - b^2v^2y^2 + v^2b^4 = 0$$

$$(u^2a^2 + v^2b^2)y^2 = v^2b^4$$

SPOR $\Rightarrow$ menovateľ $= 0$

platí $\frac{u^2}{a^2} - \frac{v^2}{b^2} + 1 = 0 \quad / a^2b^2$

$u^2b^2 - v^2a^2 = -a^2b^2$

$p' \cap Q_1 : \quad x = \frac{na^2}{mb^2} y$

$$\frac{\frac{n^2a^4}{m^2b^4}y^2}{a^2} - \frac{y^2}{b^2} - 1 = 0 \qquad | \cdot a^2b^4$$

$$n^2a^2y^2 - m^2b^2y^2 - m^2b^4 = 0$$

$$y^2(\underbrace{n^2a^2 - m^2b^2}_{=1}) = m^2b^4$$

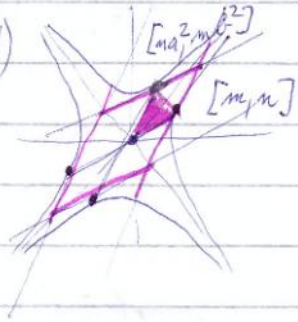
$$y^2 = m^2b^4$$

$$y_{1,2} = \pm mb^2 \qquad x_{1,2} = \frac{na^2}{mb^2} \cdot (\pm mb^2)$$

$$x_{1,2} = \pm na^2$$

$$N = [na^2, mb^2] \qquad N' = [-na^2, -mb^2]$$

a)



$$S_\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ na^2 & mb^2 & 1 \\ n & n & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} na^2 & mb^2 \\ n & n \end{vmatrix} = \frac{1}{2}\underbrace{(n^2a^2 - m^2b^2)}_{=a^2b^2} = \frac{1}{2}a^2b^2$$

$$S_{\square} = 8 \cdot S_\Delta = 4a^2b^2$$