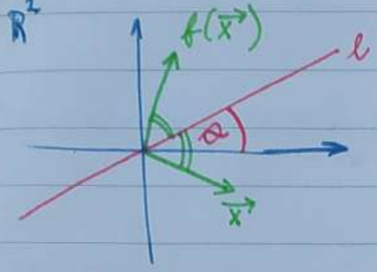
Príklad 1: Matica zrkadlenia vzhľ. na jednorozmerný podpriestor v $\mathbb{R}^2$ $\in O(2)\setminus SO(2)$.



- jednorozmerný podpriestor v $\mathbb{R}^2$ je priamka prechádzajúca cez $(0,0)$ – označme ju l
- ak l zviera s osou x uhol θ, matica zrkadlenia (osovej súmernosti) podľa priamky l je:

$$Ref(\theta) = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$ (odvodené v sada 5, pr. 3)

- chceme ukázať, že $Ref(\theta) \in O(2)\setminus SO(2)$, čiže:

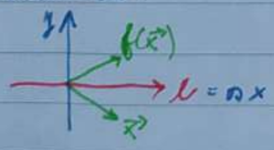
$$\left\langle \begin{array}{l} Ref(\theta) \cdot Ref^T(\theta) = I_2 \\ \det(Ref(\theta)) = -1 \end{array}\right.$$ (lebo matice $SO(2)$ majú $\det = 1$)

- počítajme:

$$Ref(\theta) \cdot Ref^T(\theta) = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

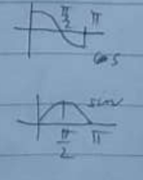
$$\det Ref(\theta) = \det \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = -\cos^2 2\theta - \sin^2 2\theta = -1 \checkmark$$

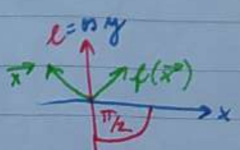
- špeciálne:



$$Ref(0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$f: (x_1, x_2) \mapsto (x_1, -x_2)$$





$$Ref\left(\frac{\pi}{2}\right) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$f: (x_1, x_2) \mapsto (-x_1, x_2)$$

Príklad 2: $M_t = \begin{pmatrix} t & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & t \end{pmatrix}$ $\quad t = ? \in \mathbb{R}$

- matica euklid. izomorfizmu

- aby matica M_t bola maticou euklidovského izomorf., musí $M_t \in O(2)$, čiže $M_t \cdot M_t^T = I_2$

$$M_t \cdot M_t^T = \begin{pmatrix} t & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & t \end{pmatrix}\begin{pmatrix} t & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & t \end{pmatrix} = \begin{pmatrix} t^2+\frac{1}{2} & 0 \\ 0 & \frac{1}{2}+t^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

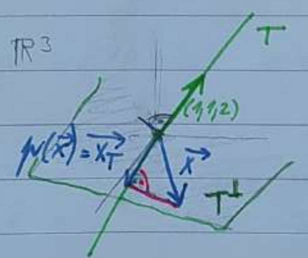
$$\Rightarrow \quad t^2 + \frac{1}{2} = 1$$

$$t^2 = \frac{1}{2}$$

$$\boxed{t = \pm\frac{1}{\sqrt{2}}} \qquad \text{čiže } M_t = \begin{cases} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = Rot\left(\frac{\pi}{4}\right) \\ \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = Rot\left(\frac{3}{4}\pi\right) \end{cases}$$

Príklad 3: $T = [(1,1,2)] \subset \mathbb{R}^3$

P - matica ortogon. projekcie $p: \mathbb{R}^3 \to \mathbb{R}^3_{(T)}$



$$\vec{x} = \vec{x_T} + \vec{x_{T^\perp}}$$

- lin. zobrazenie p je jednoznačne určené 3 dvojicami VZOR-OBR.

$(1,1,2) \mapsto (1,1,2)$ $\quad$ lebo $(1,1,2) \in T \Rightarrow p(1,1,2) = (1,1,2)$

$\vec{y} \in T^\perp \mapsto (0,0,0)$

$\vec{z} \in T^\perp \mapsto (0,0,0)$

LN trojica

- potrebujeme 2 vektory z $T^\perp$ $(\vec{y}, \vec{z})$
- hľadáme $T^\perp$: $(t_1, t_2, t_3) \in \mathbb{R}^3$ také, že $\langle (t_1,t_2,t_3), (1,1,2) \rangle = 0$

$$t_1 + t_2 + 2t_3 = 0$$

t_3, t_2 - param.
t_1 - viazaná prem.

$$\Rightarrow \quad S_H = \{(-u-2v, u, v),\ u, v \in \mathbb{R}\} = T^\perp$$

pre $u=1, v=0$ $\quad (-1,1,0)$ $\Big\} \Rightarrow T^{\perp} = [(-1,1,0),(-2,0,1)]$

$u=0, v=1$ $\quad (-2,0,1)$

- môžeme zvoliť $\vec{y} = (-1,1,0)$ a $\vec{z} = (-2,0,1)$

- potom:

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 1 & 2 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -2 & 0 & 1 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 1 & 2 \\ 0 & 2 & 2 & 1 & 1 & 2 \\ 0 & 2 & 5 & 2 & 2 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/2 & 1/2 & 1 \\ 0 & 1 & 1 & 1/2 & 1/2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & 1/6 & 1/3 \\ 0 & 1 & 0 & 1/6 & 1/6 & 1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 2/3 \end{array}\right)$$

P

- matica ortogonálnej projekcie do T je určená maticou

$$P = \begin{pmatrix} 1/6 & 1/6 & 1/3 \\ 1/6 & 1/6 & 1/3 \\ 1/3 & 1/3 & 2/3 \end{pmatrix}$$

overenie: $p(1,1,2) = (1,1,2).P = \left(\frac{1}{6}+\frac{1}{6}+\frac{2}{3}, \frac{1}{6}+\frac{1}{6}+\frac{2}{3}, \frac{1}{3}+\frac{1}{3}+\frac{4}{3}\right)$

$= (1,1,2)$ ✓ lebo $p(\vec{x}) = \vec{x}$ pre $\vec{x} \in T$

Příklad 4: $S = [(1,2,2,-1),\ (1,1,-5,3),\ (3,2,8,-7)] \subset \mathbb{R}^4$

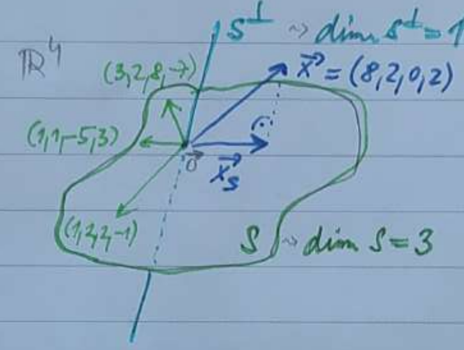
$\vec{x} = (8,2,0,2)$

$\vec{x_S} = ?$

- zajímá nás $\dim S = ?$

$$M = \begin{pmatrix} 1 & 2 & 2 & -1 \\ 1 & 1 & -5 & 3 \\ 3 & 2 & 8 & -7 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 2 & -1 \\ 0 & 1 & 7 & -4 \\ 0 & 0 & 1 & -2/3 \end{pmatrix} \Rightarrow \dim S = 3$$

$\boxed{\text{Maxima: echelon(M); stupňovitý tvar}}$



$$\vec{x} = \vec{x_S} + \vec{x_{S^\perp}}$$

$$\vec{x_S} \in S = [(1,2,2,-1),\ (1,1,-5,3),\ (3,2,8,-7)]$$

$$\vec{x_S} = \alpha_1 \underbrace{(1,2,2,-1)}_{\vec{a_1}} + \alpha_2 \underbrace{(1,1,-5,3)}_{\vec{a_2}} + \alpha_3 \underbrace{(3,2,8,-7)}_{\vec{a_3}}$$

$$\vec{x_{S^\perp}} = \vec{x} - x_S^\perp = \vec{x} - \alpha_1\vec{a_1} - \alpha_2\vec{a_2} - \alpha_3\vec{a_3}$$

- vektor $\vec{x_{S^\perp}}$ je kolmý na $\forall$ vektory z S, tj.

$$\langle \vec{x_{S^\perp}}, \vec{a_1}\rangle = 0 \ : \ \langle \vec{x} - \alpha_1\vec{a_1} - \alpha_2\vec{a_2} - \alpha_3\vec{a_3}, \vec{a_1}\rangle =$$

$$= \langle \underbrace{\vec{x}}_{(8,2,0,2)}, \underbrace{\vec{a_1}}_{(1,2,2,-1)}\rangle - \alpha_1\langle \vec{a_1}, \vec{a_1}\rangle - \alpha_2\langle \vec{a_2}, \vec{a_1}\rangle - \alpha_3\langle \vec{a_3}, \vec{a_1}\rangle = 0$$

$$10 - 10\alpha_1 + 10\alpha_2 - 30\alpha_3 = 0$$

$$\langle \vec{x_{S^\perp}}, \vec{a_2}\rangle = 0 \ : \ \langle \vec{x}, \vec{a_2}\rangle - \alpha_1\langle \vec{a_1}, \vec{a_2}\rangle - \alpha_2\langle \vec{a_2}, \vec{a_2}\rangle - \alpha_3\langle \vec{a_3}, \vec{a_2}\rangle = 0$$

$$16 + 10\alpha_1 - 36\alpha_2 + 56\alpha_3 = 0$$

$$\langle \vec{x_{S^\perp}}, \vec{a_3}\rangle = 0 \ : \ 14 - 30\alpha_1 + 56\alpha_2 - 126\alpha_3 = 0$$

- riešime sústavu: $10 - 10\alpha_1 + 10\alpha_2 - 30\alpha_3 = 0$

$$16 + 10\alpha_1 - 36\alpha_2 + 56\alpha_3 = 0$$

$$14 - 30\alpha_1 + 56\alpha_2 - 126\alpha_3 = 0$$

- riešenie: $\alpha_1 = 0 \quad \alpha_2 = 2 \quad \alpha_3 = 1$
- čiže:

$$\vec{x_s} = 0 \cdot (1,2,2,-1) + 2 \cdot (1,1,-5,3) + 1 \cdot (3,2,8,-7)$$

$$\vec{x_s} = (5,4,-2,-1)$$