

3D pen-supported geometry learning in secondary schools

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Abstract. The study of properties of three-dimensional objects and their mutual relationships often proceeds via planar projections, that is, two-dimensional representations. For some students, however, such 2D projections do not evoke an adequate mental model of the corresponding 3D object. The growing accessibility of 3D printing technologies introduces new opportunities for teaching three-dimensional geometry. This paper reports research findings demonstrating substantial benefits of employing 3D pens in the teaching of solid geometry. It also presents new instructional materials for classroom work with a 3D pen.

Keywords: 3D pen, solid geometry, teaching, education.

1 Introduction

Psychological research indicates that tactile-kinesthetic learning methods are highly effective for students. Learning is particularly effective when it includes observation, measurement, and hands-on manipulation of physical objects, an approach known as embodied learning (Hall and Nemirovsky [3], Shapiro [11]). Nevertheless, current classroom practice often provides too few opportunities for such activities. The 3D pen not only allows students to physically construct models of basic geometric solids but also enables them to draw directly in space – lines, planes, and cross-sections of solids – capabilities that were previously unavailable. Published findings suggest that using such tools improves students' conceptual understanding and retention in spatial (solid) geometry.

2 Using 3D pen in Solid Geometry Education

3D printing technology is increasingly used across diverse educational domains. In physics, students can create models of various mechanical devices. In chemistry, 3D printing can be employed to produce molecular models of chemical compounds. Medical students can also design models of organs and cells. In geography classes, students can construct topographic maps and urban planning models. In engineering and technology, students can prototype designs for technical apparatus, thereby enhancing their capacity for visualization and experimentation. Volume [2] presents a synthesis of findings from the International Symposium on 3D Printing in Mathematics Education.

2.1 The 3D Pen as an Educational Tool

A 3D pen is an innovative technology that enables the creation of spatial objects from thermoplastic materials. The pen extrudes heated plastic that cools almost immediately into a solid, stable structure, thereby allowing the freehand fabrication of three-dimensional objects. It uses a plastic filament made of polylactic acid (PLA). Compared with a 3D printer, the 3D pen is more affordable and requires no software to operate. It thus makes it possible to produce 3D models in home or school settings with minimal setup. In the instructional context, it is therefore natural to employ it for teaching solid geometry.

2.2 Related Work

Several researchers have explored the use of 3D pens in geometry instruction. Ng and Ye [8] present the first author's five-year research program on the use of 3D pens in mathematics classrooms. In the first of these studies, Ng and Sinclair [6] describe using a 3D pen to teach properties of graphs of functions. Students learned concepts such as the tangent line and slope through physical manipulation of the graph of a function. They then used a 3D pen to draw a curve together with the coordinate axes and rotated the x -axis, thereby obtaining a solid of revolution and improving their visualization of how it is formed. The study by Ng and Ferrara [4] describes a lesson design for teaching and learning the target properties of prisms, pyramids, and cross-sections of 3D solids using 3D pens alongside glass solids filled with water, where the water level represented a cutting plane. The paper by Ng, Shi, and Ting [5] compares differences in geometry learning outcomes in two technology-enhanced environments. Two groups shared an almost identical lesson procedure except for the technology used in class. The first group used 3D pens to explore the properties of prisms and pyramids, while the second group used a pre-made dynamic geometry environment (DGE). This DGE displayed a 3D solid on the computer screen, allowing students to perform basic transformations such as rotation and translation.

The findings of this five-year research program can be summarized as follows: the studies highlight the unity of mind-body and body-tool interactions in the act of making something (i.e., mathematics learning as embodied making). 3D pens, as a form of maker technology – together with embodied making – supported students' investigation, visualization, and learning of geometric relationships in significant ways. Students using 3D pens demonstrated better retention of the properties of 3D solids.

2.3 Solid Geometry with 3D Pen

All of the aforementioned studies primarily focused on younger students (aged 11–12). In contrast, this research also focuses on older students, who

often experience difficulties when solving more complex problems in solid geometry. The study of the properties and mutual relationships of three-dimensional objects often proceeds via planar projections. It is difficult for some students to realize that not all lines that intersect on paper actually have a point of intersection in the original three-dimensional space. The author of this paper supervised several completed theses aimed at developing instructional materials for teaching solid geometry using 3D pens in secondary schools.

In Redechová [10], we focused on planar cross-sections of a cube, typically taught in the seventh year (septima, age 17) of secondary school. Our objective was to demonstrate the range of polygons that can arise as the intersection of a cube with a sectioning plane (fig. 1). We deliberately selected problem statements from textbooks currently used in Slovakia, aiming to make the teaching method attractive to teachers rather than burdensome. For these examples, we developed templates for constructing 3D models of cube-plane intersections (fig. 2). Using these models, students were able to verify the correctness of their solutions and examine the results from different viewpoints. The ability to perform physical rotations proved valuable, since mental rotation is often challenging for students (Bruce and Hawes [1]).



Fig. 1: A set of 3D pen models illustrating the variety of polygons that can occur as cross-sections of a cube. Source: Redechová [10], p. 70.

The thesis by Piačková [9] aims to support students' spatial visualization in analytic geometry problems involving the computation of line-segment lengths and angles between planes and lines. This topic is typically taught in the seventh class (septima, age 17) of secondary school. The proposed tasks focus on using the dot product, the Pythagorean theorem, and the law of cosines. The templates were designed to enable the

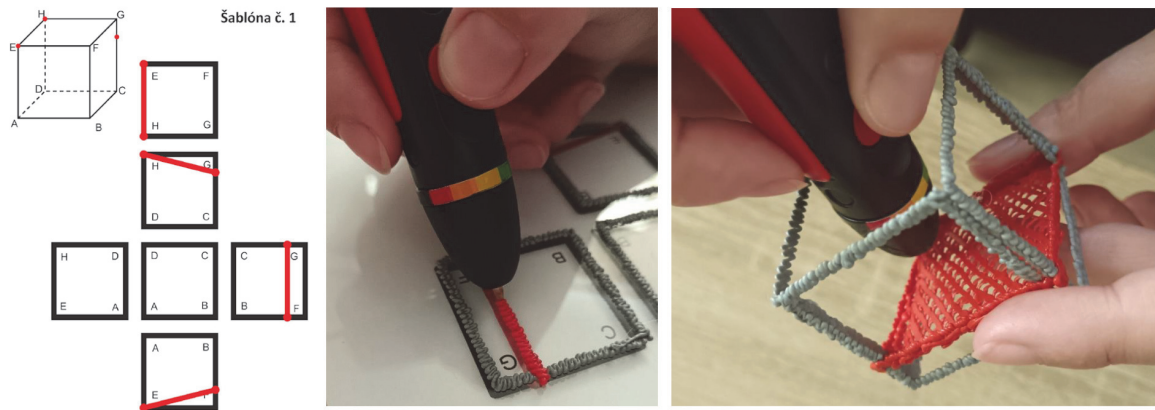


Fig. 2: Step-by-step construction of a cube–plane intersection model using a 3D pen: from the printed template to the completed 3D structure. Source: Redechová [10], pp. 67, 69, 70.

constructed models to clearly illustrate the spatial relationships among the figures whose properties students are expected to determine. Each template set includes a paper ruler, allowing students to verify the lengths of calculated line segments (fig. 3). Because paper is flexible, it is possible to perform measurements even within the interior of the cube. The measurements are not entirely precise, as the quality of the constructed model depends on the students' manual skills. Nevertheless, they are sufficient to reveal substantial deviations and to assist students in developing an awareness of the spatial position of the measured line segment. Students can situate the entire model within a coordinate system.

Trubačová [12] focused on younger students attending the second year (sekunda, age 13) of secondary school, who were learning their first projection method – oblique parallel projection. Students learn to draw basic 3D solids, such as cubes, rectangular prisms, and pyramids, in cabinet projection. Therefore, we created a template containing squares, rectangles, and triangles that can be used to build these basic solids. Thanks to the hatching on the cube's faces, the hidden edges are shown clearly as dashed lines, which is not achievable with traditional paper models students usually make at school.

Physical models enable students to explore the fundamental properties of geometric shapes through tactile and visual experience. By manipulating the models, students can literally feel what it means for edges or planes to be parallel or perpendicular, transforming abstract spatial relationships into concrete sensory experiences. This multimodal perception greatly supports the development of accurate mental representations of solids. The perception of a cube's face depends on the viewing angle (by sight one perceives a parallelogram; by touch, a square). Building these

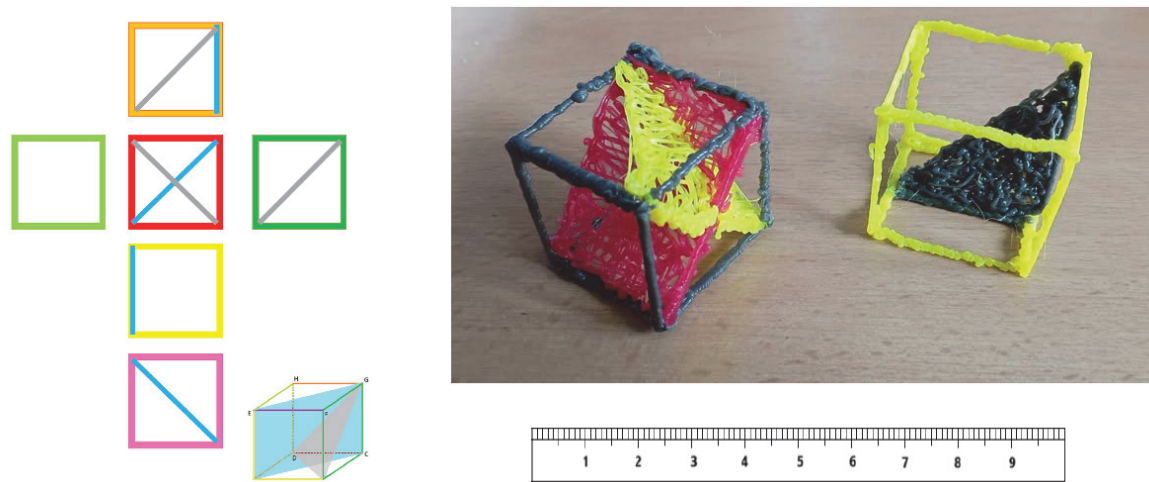


Fig. 3: Classroom models created with a 3D pen: the left model, together with its corresponding template, illustrates the perpendicularity of two planes, while the right model, together with a paper ruler, serves to verify calculated lengths and areas using the Pythagorean theorem. Source: Piačková [9], pp. 58, 71, 78.

mental connections is very important at this age (fig. 4).

The thesis also includes proposed communication scenarios that represent model dialogues between the teacher and the student, designed as a methodological framework for conducting mathematics lessons that employ 3D pen technology. These scenarios are structured to foster guided discovery and student reflection through question sequences that encourage independent formulation of geometric concepts, relationships, and projection rules. They include progressively graded interactions – from motivational and situational prompts to analytical and verification-oriented questions that reinforce and apply newly acquired knowledge during task completion. The dialog structure simultaneously strengthens elements of the constructivist approach, promotes students' reasoning and argumentation skills, and creates space for mutual discussion and cooperative learning.

2.4 Application in the Classroom

All three authors of [10], [9], and [12] were teaching mathematics while studying, which enabled them to test the designed teaching aids directly in classrooms.

In Redechová [10], two parallel classes were involved in the experiment. Each class was assigned general tasks for spatial imagination. After analyzing the results of these tasks, the class that achieved weaker results was identified. This class (11 students) was selected as the experimental group in which teaching using a 3D pen was implemented. In the second class,

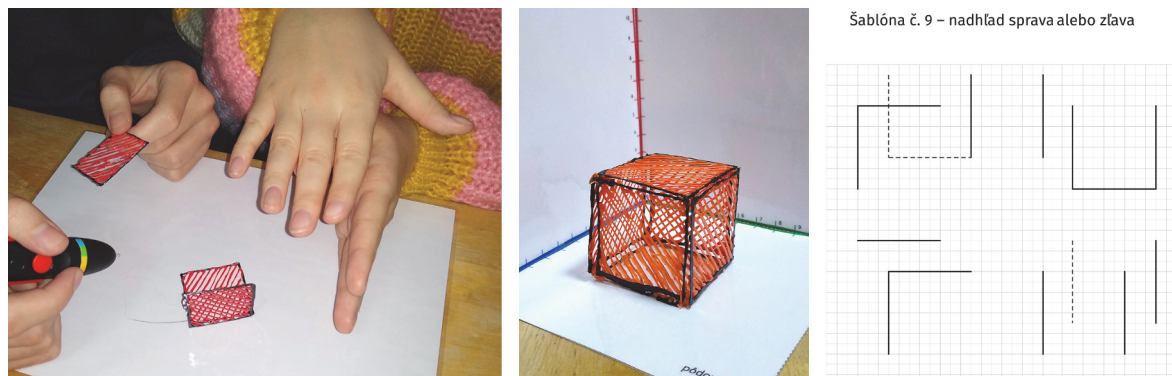


Fig. 4: Classroom activity on cabinet projection: on the left, students construct a cube model and use hand gestures to indicate the perpendicularity of its faces; in the center, the completed cube is placed in a coordinate system – hatched faces make it possible to distinguish visible and hidden edges; on the right, the worksheet template on which students supplemented the diagram with visible (solid) and hidden (dashed) edges to produce the cube's cabinet projection (left or right view), using the constructed model for reference. Source: Trubačová [12], pp. 36, 44, 64.

designated as the control group (12 students), teaching was conducted traditionally, using paper-based drawing. After the end of the teaching, students in both groups were given further tasks to develop their spatial imagination, including a written test. Although the sample of students was too small for a statistically relevant assessment of the effectiveness of teaching with 3D pens compared to standard methods, the experimental group demonstrated greater attention, anticipation, and active engagement during lessons.

In Piačková [9], 18 students worked together in groups of three and took turns using the 3D pen to construct geometric models. According to their responses in the post-lesson questionnaire, they particularly appreciated the opportunity to verify their calculated results by directly comparing them with the physical model. The tangible representation made it easier for them to identify geometric properties, such as which of the constructed triangles were right-angled, although they subsequently had to provide a formal mathematical justification. The teacher's observations also revealed that even students who are usually passive during math lessons were actively involved in this collaborative environment. Working in groups and using the 3D pen stimulated motivation, encouraged students to participate in problem solving, explore alternative approaches, and engage in independent learning – often without realizing it. Based on the observations, the new approach was perceived as more attractive and innovative compared with traditional teaching methods and was a good

complement to paper-based constructions.

The experimental teaching in Trubačová [12] was implemented in two student groups comprising 12 and 15 seventh-grade learners (equivalent to the second year of secondary school). Throughout the sessions, a high level of engagement was observed, accompanied by spontaneous nonverbal expression of understanding. Students unconsciously used hand gestures to represent geometric concepts – for example, illustrating a cube's face with the palm of the hand, indicating parallel and perpendicular lines with appropriately positioned fingers, or marking points with fingertips. These gestures served as clear indicators of internalized comprehension of spatial relationships. The use of two-color filaments further enhanced visual clarity, enabling students to distinguish geometric elements such as edges and faces more easily. Many students expressed enthusiasm for working with the 3D pen, noting that they preferred creating geometric figures in this way rather than with a pencil, as it allowed them to physically touch and examine their 'drawings' from multiple perspectives. Even those who struggled to complete a full model – whether due to breakage or other technical issues – remained engaged by borrowing models from classmates and continuing to solve related tasks collaboratively. Students were evidently more focused and motivated when working with objects they had created themselves than when merely viewing teacher-prepared models.

3 Conclusion

Across all conducted studies, the use of 3D pen technology in geometry instruction has demonstrated strong potential to enhance students' spatial reasoning, conceptual understanding, and engagement. The combination of tactile manipulation and visual observation enables learners to connect abstract geometric principles with concrete experience, thereby reducing misconceptions and improving the accuracy of spatial visualization. Collaborative group work with 3D pens promotes active participation, self-directed learning, and peer-to-peer explanation, even among typically passive students. Teachers have also emphasized the accessibility and intuitive nature of the tool, which requires minimal technical preparation while offering high didactic value. The results collectively suggest that integrating 3D pen activities into mathematics education can effectively bridge the gap between theory and practice. Future research should focus on long-term impacts and evaluation of the effectiveness of using the proposed teaching aids across multiple schools. We plan to develop additional teaching materials for other areas of solid geometry and to create a platform where these materials will be freely available for download, together with usage instructions. All of these measures are aimed at making the study of geometry and methods of descriptive geometry more attractive.

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